ANALYSIS OF FREEWAY ON-RAMP CAPACITIES BY MONTE CARLO SIMULATION AND QUEUING THEORY

OCT. 1964

NO. 27

by

R. F. DAWSON
and
H. L. MICHAEL

Joint Highway Research Project

PURDUE UNIVERSITY
LAFAYETTE INDIANA

Technical Paper

ANALYSIS OF FREEWAY ON-RAMP CAPACITIES BY MONTE CARLO

SIMULATION AND QUEUING THEORY

TO: K. B. Woods, Director
Joint Highway Research Project

October 28, 1964

FROM: H. L. Michael, Associate Director
Joint Highway Research Project

Project: C-36-17CC
File: 8-4-29

The attached Technical Paper entitled "Analysis of Freeway On-Ramp Capacities by Monte Carlo Simulation and Queuing Theory" has been authored by Messrs. R. F. Dawson and H. L. Michael. It is a summary of the Final Report of a similar title which was submitted to the Board at the September 18 meeting.

The paper has been offered to the Highway Research Board for presentation and publication at the 1965 Annual Meeting in January 1965. It is presented to the Board for release for such presentation and publication if accepted.

Respectfully submitted,

Harold L. Michael

Harold L. Michael, Secretary

HLM:bc

Attachment

Copy:

F. L. Ashbaucher
J. R. Cooper
W. L. Dolch
W. H. Goetz
W. L. Grecco
F. F. Havey
F. S. Hill
G. A. Leonards
J. F. McLaughlin
F. B. Mendenhall
R. D. Miles
R. E. Mills
J. C. Oppenlander
W. P. Privette
M. B. Scott
J. V. Smythe
E. J. Yoder

Digitized by the Internet Archive
in 2011 with funding from
LYRASIS members and Sloan Foundation; Indiana Department of Transportation

http://www.archive.org/details/analysisoffreewa00daws

Technical Paper

ANALYSIS OF FREEWAY ON-RAMP CAPACITIES BY MONTE CARLO SIMULATION AND QUEUING THEORY

by

Robert F. Dawson
Research Assistant

and

Harold L. Michael
Associate Director

Joint Highway Research Project

Project: C-36-17CC

File: 8-4-29

Purdue University

Lafayette, Indiana

October 28, 1964

ANALYSIS OF FREEWAY ON-RAMP CAPACITIES BY MONTE CARLO SIMULATION AND QUEUING THEORY

BY

Robert F. Dawson

and

Harold L. Michael

Synopsis

This research paper is concerned with the analysis of the capacities of three different freeway on-ramp designs--on ramps with no acceleration lane and stop-sign control, on-ramps with no acceleration lane and yield-sign control, and on-ramps with an acceleration lane and no sign control. The study includes the development of criteria for defining both possible and practical capacities, the development of a deterministic queuing model for predicting possible capacity, the development of a Monte Carlo simulation model for the study of ramp flow under varying traffic conditions, the evaluation of vehicle delays and queue lengths incurred by on-ramp vehicles for various combinations of ramp and shoulder-lane traffic volumes, and the evaluation of possible and practical on-ramp capacities for the three different ramp designs.

ABRIDGEMENT

Dawson, Robert F., and Michael, H. L., "Analysis of Freeway on-Ramp Capacities by Monte Carlo Simulation and Queuing Theory". Manuscript contains 73 pages of which 20 are illustrations and 16 are tables.

Descriptors: Freeway on-ramp capacity; Monte Carlo simulation; queuing theory; on-ramp traffic characteristics; Dawson, Robert F., Michael, H. L.

This research was concerned with the analysis of the capacities of three different freeway on-ramp designs--namely, on-ramps with no acceleration lane and stop-sign control, on-ramps with no acceleration lane and yield-sign control, and on-ramps with an acceleration lane and no sign control. The study included the development of criteria for defining both possible and practical capacities, the development of a deterministic queuing model for predicting possible capacity, the development of a Monte Carlo simulation model for the study of ramp flow under varying traffic conditions, the evaluation of vehicle delays and queue lengths incurred by on-ramp vehicles for various combinations of ramp and shoulder-lane traffic volumes, and the evaluation of possible and practical on-ramp capacities for the three different ramp designs.

Initial research efforts were concerned with the development of descriptors of the ramp situation. The distribution of headways between ramp vehicles was described by a hyper-exponential model. All ramp vehicles were assumed to enter the ramp system at a constant speed, controlled by the critical geometry of the area rather than by traffic. Ramp vehicle behavior in the system was defined by four factors--the spacing relationship with the preceding vehicle, acceleration-deceleration

capabilities, the availability of gaps in the shoulder lane, and distributions describing gap-acceptance phenomena.

Shoulder-lane headways were described by a shifted-exponential model. Each shoulder-lane vehicle was assigned a speed upon entry into the system that was only dependent upon the volume of traffic in the shoulder lane. It was further assumed that the shoulder-lane vehicles proceeded through the ramp area at the speeds and headway spacing assigned at generation, without any interference from ramp traffic.

The various traffic descriptors were expressed in the mathematical mode and assembled for analysis into two different types of models--a deterministic queuing model for the analysis of possible ramp capacity, and a Monte Carlo simulation model for the analysis of practical capacity.

Because both models were constructed in the mathematical mode they were readily programmed for computer solution. The programs were coded in the FORTRAN IV and MAP languages for the IBM 7090/7094 System and were run on an IBM 7090.

The results obtained from the queuing-model analysis were reported in graphical form. The possible capacities of each of the three ramp designs were plotted as functions of shoulder-lane volume. Delay and queuing characteristics for a wide range of ramp and shoulder-lane volume combinations were obtained from the simulator. Practical capacities were defined for each of the three ramp designs by analyzing the delay characteristics relative to the criteria established for practical capacity in the definition of the same. Queue storage requirements on the ramp were found by an analysis of queuing characteristics at practical-capacity volume levels.

INTRODUCTION

In recent years thousands of miles of freeway-type highways have been constructed to provide for the safe, convenient and efficient transportation of persons and goods. Access to these high-type traffic-carrying facilities is provided by on-ramps that are designed to merge ramp traffic into the high-speed, high-volume traffic stream. The efficiency of traffic movement on freeways, and the extent to which the potential capacity of freeways can be realized, depends in part on the adequacy of the access facilities. Improperly designed entrances limit the volume of traffic that can use an expressway and generate congestion that often extends back onto the local system.

Purpose and Scope of Study

The purpose of this study was four-fold:

1. To develop criteria for defining possible and practical on-ramp capacities;

2. To develop general models for the analysis of flow through the merge area;

3. To evaluate vehicle delays and queue lengths that are incurred by on-ramp vehicles for various combinations of freeway and ramp volumes; and

4. To define the possible and practical capacities of freeway on-ramps for each of three design-control situations.

Freeway on-ramp capacity is controlled at one or more of three locations along the typical ramp. These locations are-- (1) the entrance to the ramp from the local system or another freeway, (2) the ramp proper, and/or (3) the merge area at the freeway terminal of the ramp. This

study was devoted to an analysis of the latter location, the merge area at the freeway terminal, as it is more commonly the restricting element of the ramp.

Only ramps with geometric configurations such that on-ramp merge maneuvers are not compounded with off-ramp diverge maneuvers were considered. Thus the analysis is pertinent to the on-ramps of diamond interchanges and to the outer-loop connectors of cloverleaf interchanges. The typical ramp-terminal designs and controls which are used on existing freeways were analyzed and compared--no acceleration lane with stop-sign control, no acceleration lane with yield-sign control, and an acceleration lane with no sign control. The layouts assumed for these control situations are shown in Figure 1 for no acceleration lane with stop-or yield-sign control and in Figure 2 for an acceleration lane with no sign control.

The conduct of a field study of the scope proposed was impractical with respect to both cost and time. In addition data from numerous traffic studies of existing access facilities located throughout the country were already available. The existence of these data, plus the availability of a modern high-speed digital computer, suggested the development of simulation and queuing models for analyzing ramp capacity.

CRITERIA FOR ON-RAMP CAPACITY

The term "capacity," as it is applied to highway traffic facilities, is not uniquely descriptive. In general, it pertains to the ability of a facility to accommodate traffic; but without some criteria indicative of the level of performance associated with a volume of flow, the statement of a numerical capacity limit is incomplete.

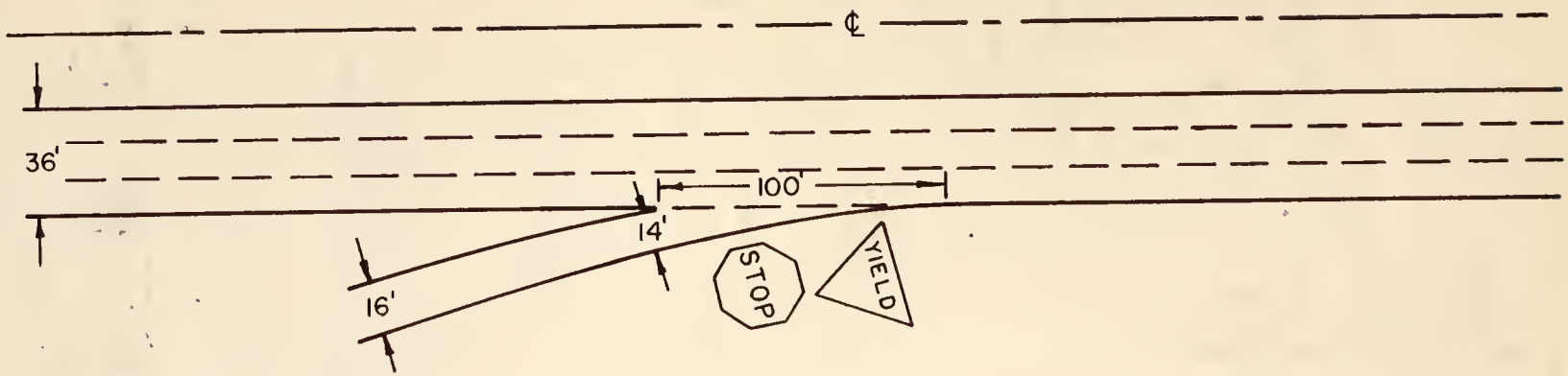


FIG. 1 - TYPICAL ON-RAMP WITHOUT ACCELERATION LANE

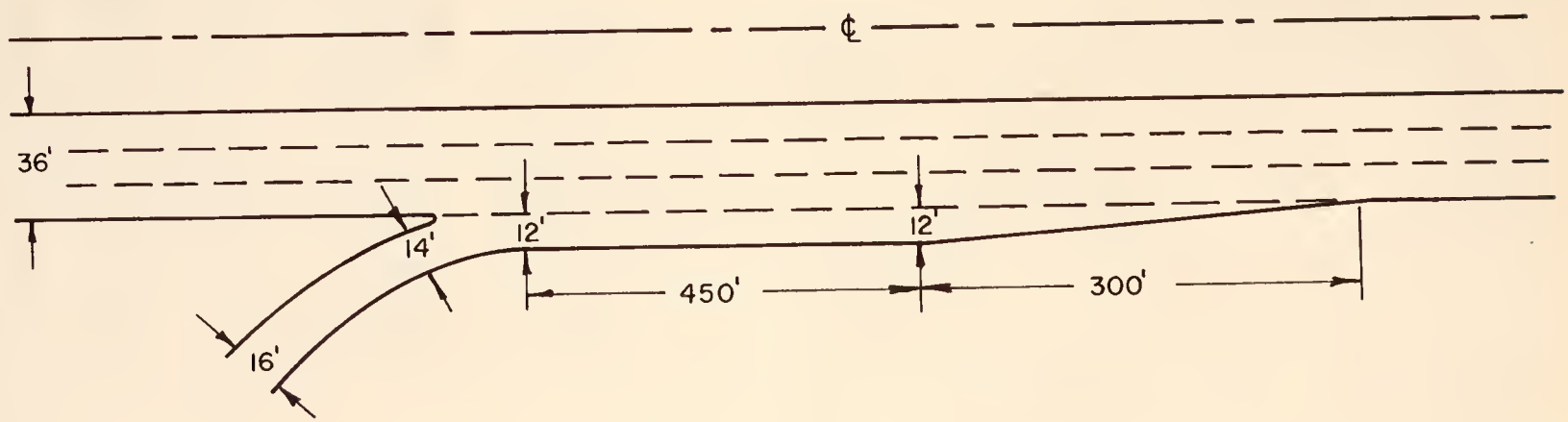


FIG. 2 - TYPICAL ON-RAMP WITH ACCELERATION LANE

Two variables, commonly considered as yardsticks of performance, are vehicle delay and queue length. Vehicle delay can be expressed in terms of the average delay incurred by a vehicle for various combinations of ramp and freeway volumes, or as the probability that delay exceeds some established level. Queuing characteristics can be defined by the mean queue length, or in terms of some percentile value such as the 85th, 90th, or 95th percentile queue length. Although definite limits should be established for the delay indices, there is no reason to establish numerical limits for general application in the case of queues. A design engineer should merely use defined queuing characteristics to establish storage requirements for ramp traffic, when the ramp is operating at a capacity level established relative to delay characteristics.

In an attempt to establish a uniform capacity concept the American Association of State Highway Officials (1)* adopted definitions of highway capacity at two levels of performance. These definitions follow:

> "Practical Capacity represents the maximum number of vehicles that can pass a given point on the lane or roadway during one hour under the prevailing roadway and traffic conditions, without unreasonable delay or restriction to the driver's freedom to maneuver."

> "Possible Capacity is the maximum number of vehicles that can pass a given point on a lane or roadway during one hour under the prevailing roadway and traffic conditions, regardless of their effect in delaying drivers and restricting their freedom to maneuver."

Although these definitions were intended for uninterrupted traffic facilities, the rationale can be applied to ramp situations. In addition, the Highway Capacity Manual (2) definitions for capacities of signalized

---

* Numbers in parentheses refer to entries in the List of References.

intersections suggested an index for describing reasonable delay. These definitions state:

> "The Practical Capacity of an intersection approach under signal control is the maximum volume that can enter the intersection from that approach during one hour with most of the drivers being able to clear the intersection without waiting for more than one complete signal cycle."
>
> "Possible Capacity is the maximum number of vehicles that can actually be accommodated under the prevailing conditions with a continual backlog of waiting vehicles."

Two modifications to the practical-capacity definition were necessary before it was applicable to the ramp situation. The first modification involved the number of drivers being delayed. The qualitative index "with most of the drivers being able to clear the intersection" was replaced by the quantitative index "with 85 percent of the drivers being able to clear the intersection." The second modification involved the length of delay incurred by the drivers. Since signals are not commonly used for traffic control on on-ramps the time unit "one signal cycle" was replaced by an approximately equivalent time period of "60 seconds." The definition proposed for the practical capacity of a freeway on-ramp is as follows:

> The practical capacity of a freeway on-ramp is the maximum volume of vehicles that can enter the through highway during one hour with 85 percent of the drivers being able to leave the ramp without being delayed for more than 60 seconds.

The definition of the possible capacity of a signalized intersection was applicable to the ramp situation without modification.

DESCRIPTORS OF THE RAMP SITUATION

The many variables involved in the operation of the ramp area traffic system can be classified under five headings--roadway characteristics, vehicle characteristics, driver characteristics,

traffic and environmental conditions, and rules of operations.

Roadway Characteristics

Geometric Layout

The two on-ramp geometric layouts analyzed in this study were presented in Figures 1 and 2. The dimensions assumed for the no acceleration-lane design were based on a survey of plans of existing facilities. Although no exact locations were defined for the entry point to the system, for the stop-line, or for the point of entry into the shoulder lane, the proposed design was adequate to provide 108 feet for deceleration from ramp speed to a stop at the stop-line. In addition the assumption was made that the vehicle traveled a distance of 92 feet from the stop-line to the point of entry into the shoulder lane.

The dimensions for the acceleration-lane design were based on a survey of recommended ramp designs. With a 450-foot acceleration-lane proper and a 300-foot taper, ramp vehicles had approximately 500 feet of acceleration distance available before encroaching on the shoulder lane. This distance was just adequate to provide for acceleration from a stop at the ramp nose to the maximum average shoulder-lane speed. Again the ramp geometry was adequate for the driver to decelerate from the ramp speed to a stop at the ramp nose with a comfortable rate of deceleration when such a maneuver was deemed necessary.

Traffic Control

Three spearate traffic-control conditions were analyzed. Both stop-sign and yield-sign control devices were established on the no

acceleration-lane layouts; no sign control was established on the acceleration-lane layout. Results reported from previous research studies indicated that stop-sign control devices on ramps often function as yield signs or as a cross between a stop sign and a yield sign. For the purposes of this study each device was required to function in compliance with the regulations defined in the Manual on Uniform Traffic Control Devices.

Vehicle Characteristics

All of the vehicles traversing the ramp system, whether on the ramp or on the shoulder lane, were assumed to have the geometric and operating characteristics of passenger cars. Overall length was established at 16.5 feet, the approximate average for all passenger cars, although this is considerably shorter than the AASHO defined P design vehicle (1).

In addition each vehicle was assigned constant acceleration and deceleration potentials of five and six miles per hour per second, respectively. In reality acceleration and deceleration rates have distributions that are functions of the vehicle, the driver, the roadway, and the environment; but because of inadequate data and for simplicity, these variables were defined as constant vehicle characteristics.

Driver Characteristics

PIEV Time

Although the driver is probably the most complex and certainly the dominant element in the ramp traffic system he was modeled as a

relatively simple machine with a capability for completing the "PIEV" process in 1.5 seconds. Although it is known that perception, intellection, and volition time requirements, under emotional stress, vary among and within drivers, as well as among situations, lack of information on this distribution led to the selection of the above average, and hopefully, representative constant time.

Minimum Time and Space Clearances

The minimum time and space clearances that a driver demands as a buffer between himself and a lead vehicle are undoubtedly closely related to his PIEV time. Various minimum clearances were established. A driver normally would not position his vehicle with less than five feet of clearance to a leading vehicle, and he would not move into a shoulder-lane gap behind a shoulder-lane vehicle with a time clearance of less than 0.5 seconds. Minimum clearance time for ramp vehicles following a leading ramp vehicle through the system varies with the ramp design and the type of traffic control; the minimum was established at 2.0 seconds with no acceleration lane and yield-sign control, whereas it was set at 1.8 seconds with an acceleration lane and no control. In the latter case sudden, abrupt stops are less likely to occur. No limits were established for the stop-sign condition as the minimum clearance to a leading ramp vehicle never controls. Minimum headway spacings in the moving ramp and shoulder-lane streams were also defined, but these are discussed under traffic and environmental conditions.

Gap Acceptance

Gap acceptance was the final driver characteristic to be modeled.

From several studies of this phenomenon conducted in recent years (7, 8, 9, 10, 11, 16, 18, 19, 20), it was possible to develop two families of gap-acceptance models--one for on-ramps without acceleration lanes and one for on-ramps with acceleration lanes. In both cases distinction was made between gap acceptance by stopped first-in-line vehicles and gap acceptance by vehicles that were moving as they passed the first-in-line position.

In the case of stop-sign control on on-ramps without acceleration lanes all vehicles were assumed to stop in the first-in-line position before departing from the ramp system. The gap-acceptance model for this condition, derived from data collected by Pearson and Ferreri (16) on the Schuylkill Expressway, was of the mathematical form,

$$Pr(Acpt) = 1 - e^{-\left(\frac{t-t_{min}}{\bar{t}-t_{min}}\right)},$$

where:

$Pr(Acpt)$ = probability of accepting a gap of length, t;

t = any gap greater than t_{min};

t_{min} = minimum acceptable shoulder-lane gap; and

$\bar{t}$ = the average acceptable shoulder-lane gap.

With appropriate parameter substitutions the model was written as,

$$Pr(Acpt) = 1 - e^{-\frac{t - 3.3}{6.5 - 3.3}}$$

or

$$Pr(Acpt) = 1 - e^{(1.021 - 0.313t)}$$

Of course this same model holds for stopped first-in-line vehicles departing from a ramp with no acceleration lane and yield-sign control.

A similar model was proposed by Weiss and Maradudin (18) for vehicles that do not stop in the first-in-line position, as would be the case with yield-sign control. This model, written as,

$$Pr(Acpt) = 1 - e^{-\frac{t - 2.0}{5.0 - 2.0}}$$

or

$$Pr(Acpt) = 1 - e^{(0.667 - 0.333t)} ,$$

was accepted as a suitable descriptor of gap acceptance at on-ramps with no acceleration lane and yield-sign control.

Gap-acceptance models for on-ramps with acceleration lanes and no sign control were developed from data collected by the Texas Transportation Institute (8). Seperate models were defined for vehicles that depart from the system after stopping in the first-in-line position and for vehicles not required to stop in the first-in-line position. Both models follow the mathematical form,

$$Pr(Arpt) = \ln\left(\frac{t}{t_{min}}\right) * \frac{1}{\ln\left(\frac{t_{max}}{t_{min}}\right)} ,$$

where:

t = any gap length between the limits of t_{min} and t_{max};

t_{min} = minimum acceptable gap; and

t_{max} = minimum gap length for which probability of acceptance is one.

With appropriate parameter substitutions the models for gap acceptance after a stop and with no stop are respectively:

$$Pr(Acpt) = \ln\left(\frac{t}{2.50}\right) * \frac{1}{\ln\left(\frac{8.00}{2.50}\right)}$$

$$= 0.787 + 0.859 \ln(t);$$

and

$$Pr(Acpt) = \ln\left(\frac{t}{1.00}\right) * \frac{1}{\ln\left(\frac{4.00}{1.00}\right)}$$

$$= 0.722 \ \ln(t).$$

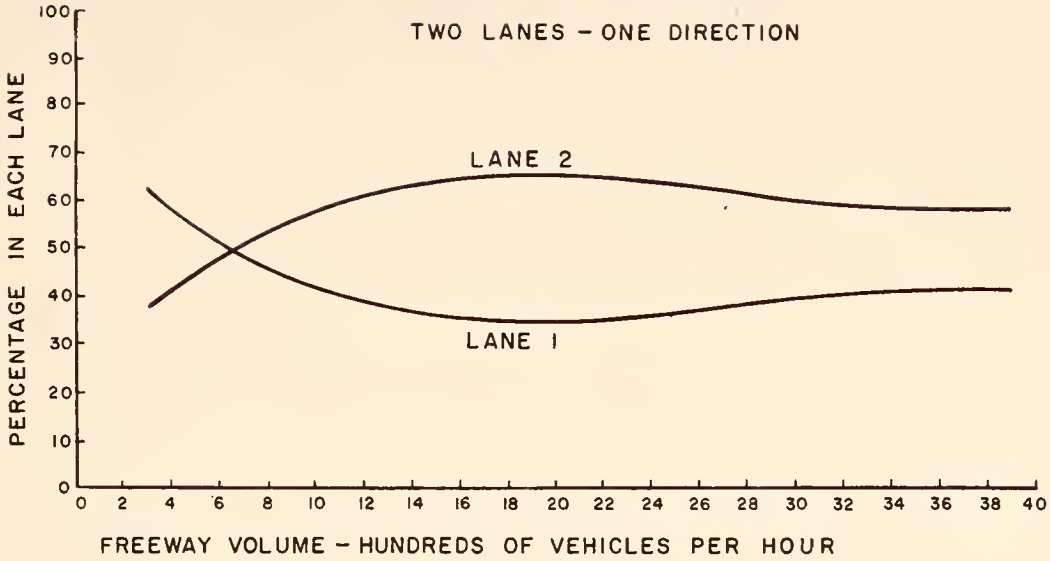
Traffic and Environmental Characteristics

Traffic and environmental characteristics are presented together as they are closely related. Changes in environmental conditions such as weather, lighting, roadside development, etc. tend to modify traffic characteristics. For the purposes of this research environmental conditions were assumed ideal without any statement as to meaning of "ideal."

Traffic Distribution Between Lanes

Numerous lane-distribution studies have been conducted. Most of these studies (2, 7, 10, 11, 12, 16) reported traffic distribution between lanes as a function of total one-direction freeway volume only. Two recent studies--one published by Moskowitz and Newman (13), and the other published by Hess (4)--reported that lane distribution is dependent upon such variables as number of freeway lanes, total freeway volume, distance upstream to last off-ramp, distance downstream to next off-ramp, traffic volume off at last off-ramp, traffic volume off at next off-ramp and ramp traffic on the ramp under consideration.

After a thorough review of the available data a decision was made to use Hess's models which were derived from data obtained in a comprehensive, nationwide ramp-capacity study under the sponsorship of the Bureau of Public Roads. The results are presented in two forms for both the four-lane and six-lane freeways. In Figure 3 lane distributions,



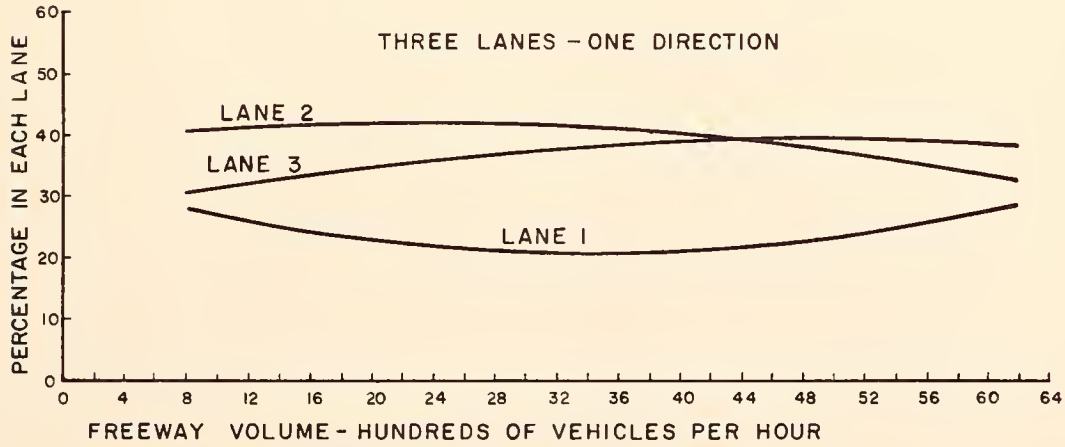


FIG. 3 - DISTRIBUTION OF TRAFFIC VOLUME BETWEEN LANES IN ONE DIRECTION AT APPROACH TO RAMP

given as a function of numbers of lanes and of total one-direction volume, can be used for approximate estimates of shoulder-lane volume. When more refined estimates are desired the following equations can be used:

1. Four-lane freeways

$$V_s = -1.21 + 0.244V_s - 0.085V_{ur} + 640_{dr}/D_{dr},$$

2. Six-lane freeways

$$V_s = 55 + 0.363V_f - 0.184V_r + 0.022D_{dr} + 0.057V_{dr}$$

where:

V_s = shoulder-lane volume (vph),

V_f = total one-way freeway volume (vph),

V_r = ramp volume (vph),

V_{ur} = volume on adjacent upstream off-ramp (vph),

V_{dr} = volume on adjacent downstream off-ramp (vph), and

D_{dr} = distance to adjacent downstream off-ramp.

The multiple R^2's for these four-lane and six-lane models were 0.92 and 0.80, respectively; the coefficients of variation were reported as 0.086 and 0.134.

Headway-Vehicle Generators

Several probabilistic models are available as descriptor of headways in traffic streams. The more common ones are the negative-exponential distribution (3), the shifted-exponential distribution, the hyper-exponential distribution (6, 17), and a modified binomial distribution (3,20). For the purposes of this study the shifted-exponential model was used to describe headways in the shoulder-lane

stream, and the hyper-exponential model was used to describe ramp headways.

The shifted-exponential model is described by the mathematical model,

$$P\ (h \geq t) = e^{-\left(\frac{t-D}{\bar{t}-D}\right)},$$

where:

$P\ (h \geq t)$ = probability that a headway is equal to or greater than t,

t = any time,

$\bar{t}$ = average headway in stream,

= 3600/hourly volume, and

D = minimum allowable headway in the stream.

By trial-and-error process D-values were defined for various shoulder-lane volumes to effect an apparent good fit to the headway curves for multi-lane traffic streams given in the Highway Capacity Manual (2). The varying D-values were described as a function of the shoulder-lane volume by the expression,

$$D = 0.31 + .0001V_s.$$

The hyper-exponential headway distribution used to describe the ramp traffic stream was originally proposed by Schuhl (17), but the necessary statistical evaluation was performed by Keil (6). This distribution is based on the theory that a traffic stream is made up of two populations of moving vehicles--a restrained population and a free-moving population--each with its own headway distribution. The overall headway distribution is therefore defined by the expression,

$$P\ (h \geq t) = (1 - \alpha)\, e^{-\left(\frac{t - \Delta_1}{T_1 - \Delta_1}\right)} + e^{-\left(\frac{t - \Delta_2}{T_2 - \Delta_2}\right)},$$

where:

α = the proportion of the traffic stream in the restrained population,

$1 - \alpha$ = the proportion of the traffic stream in the free-moving population,

T_1 = average headway of the free-moving population,

T_2 = average headway of the restrained population,

Δ_1 = the minimum allowable headway of the free-moving population, and

Δ_2 = the minimum allowable headway of the restrained population.

Kell evaluated the parameters of this model on a two-lane urban street on which there was negligible passing opportunity. Since the characteristics of a one-lane ramp are not unlike those of the directional channels of an urban street, Kell's model was assumed to afford an adequate description of headways in a ramp stream.

Speed Models

Although speeds are known to follow approximately normal distributions in freeway flow, this variable was described by much simpler models for speed in the ramp and shoulder-lane streams. Ramp entrance speed was assigned a constant value of 30 miles per hour, on the assumption that ramp geometry governs speed regardless of traffic conditions. Shoulder-lane speeds were estimated by an equation approximating two models developed at the Midwest Research Institute (7). The models reported by this group were:

$$SP = 51.062 - 0.0085V_s$$

and

$$SP = 53.703 - 0.0077V_s \quad ,$$

where:

SP = shoulder-lane speed in the ramp vicinity, and

V_s = shoulder-lane volume (vph).

The values for the coefficients of determination were only 0.337 and 0.545 respectively; whereas the coefficients of variation were 0.18 and 0.16. The model actually used in this study,

$$SP = 52.0 - 0.008V_s ,$$

was an approximate average of those given above.

Rules of Operation

The rules of operation embody a queuing discipline and rules for the driver-vehicle under various traffic conditions.

Queuing Discipline

The definition of an appropriate queuing discipline is relatively simple. The geometry of the ramp area is such that service is provided to ramp traffic on a first-come, first-served basis. That is, no trailing vehicle can preempt service priority and pass a leading vehicle to accept a gap in the shoulder lane.

Vehicle and/or Driver Behavior

A driver arriving at the entry point to the ramp system should immediately decide his course of action. If there is no acceleration lane and stop-sign control exists, the driver's decision should be to decelerate to a stop. Since there were 108 feet available between the point of entry into the ramp system and the stop-line in this study, this maneuver could be effected at a comfortable rate of deceleration. All drivers were assumed to utilize the same acceleration rates and require the same minimum time and space clearances previously established.

In the cases of no acceleration lane with yield-sign control and an acceleration lane with no sign control, the driver's decision process

at the point of entry into the system is somewhat more complex. Upon passing this entry point he should evaluate both shoulder-lane and ramp traffic conditions and establish his course of action. His decision may be to stop upon, or before, reaching the stop-line; or his decision may be to proceed through the ramp area and into the shoulder lane. To arrive at the latter decision the driver has to project the positions and speeds of all other vehicles in the system, as well as his own, to the most critical point in both time and space. His decision to stop or proceed is based entirely upon gap acceptance. He may determine the acceleration-deceleration pattern, within the capabilities established for his vehicle, that will maximize his probability of accepting a gap. It is probable, however, that a driver will not follow the speed pattern that maximizes the gap available to him (maximizing the gap maximizes the probability of accepting the gap), but he undoubtedly considers the best situation that he can create for himself before making a decision.

Some restrictions were necessary, however, to control this complex situation so that a model could be developed; and it was assumed that the driver would not stop on the acceleration lane or at any point on the shoulder-lane side of the stop-line. It was also assumed that in every instance he would stop at the stop-line if an alternate course of action would result in a speed at any point on the shoulder-lane side of the stop-line that would be lower than the speed that would be attained at that point during acceleration from a stop at the stop-line.

MACRO MODELS FOR ON-RAMP CAPACITY

The ramp situation was described in micro detail in the previous

section. Every important aspect of the ramp traffic system was formulated as a descriptive behavior model or as a rule of operation. This chapter is devoted to the development of a macro framework in which to assemble the micro models as functioning systems. The components were first pulled together into queuing models for analyses of the possible capacities of each of the various design-control combinations--no acceleration lane with stop sign control, no acceleration lane with yield-sign control, and an acceleration lane with no sign control. Following this, Monte Carlo simulations were constructed for analysis of the practical capacities of these ramp configurations.

Queuing Model for Possible Capacity

By definition possible capacity is the maximum number of vehicles that can be accommodated with a continual backlog or queue of vehicles. Whenever the opportunity occurs for n vehicles to enter the shoulder-lane stream there must be at least n vehicles queued on the ramp to utilize this capacity potential. Although the delay associated with such a traffic condition may be unreasonable, it was not included in the capacity analysis.

A single general queuing model was written to describe the possible capacity of freeway on-ramps regardless of the design, and regardless of the type of control, if any. This model is as follows:

$$C_{x_1} = V_c \left[0 \left\{ \begin{array}{l} (P(t_0)) \\ + (P(t_1) \cdot R(T_1)) \\ + (P(t_2) \cdot R(T_2)) \\ \circ \\ \circ \\ \circ \end{array} \right. \right.$$

$$+ (F(t_n) \cdot R(\tau_n))\Big\}$$

$$+ \gamma \Big\{ (F(t_1) \cdot R(\tau_1))$$

$$+ (F(t_2) \cdot R(\tau_2) \cdot R(\tau_1))$$

$$+ (F(t_3) \cdot R(\tau_3) \cdot R(\tau_2))$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$+ (F(t_n) \cdot R(\tau_n) \cdot R(\tau_{n-1}))\Big\}$$

$$+ \gamma \Big\{ (F(t_2) \cdot R(\tau_2) \cdot R(\tau_1))$$

$$+ (F(t_3) \cdot R(\tau_3) \cdot R(\tau_2) \cdot R(\tau_1))$$

$$+ (F(t_4) \cdot R(\tau_4) \cdot R(\tau_3) \cdot R(\tau_2))$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$+ (F(t_n) \cdot R(\tau_n) \cdot R(\tau_{n-1}) \cdot R(\tau_{n-2}))\Big\}$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$+ (n-1) \Big\{ (F(t_{n-1}) \cdot R(\tau_{n-1}) \cdot R(\tau_{n-2}) \cdot \ldots \cdot R(\tau_2) \cdot R(\tau_1))$$

$$+ (F(t_n) \cdot R(\tau_n) \cdot R(\tau_{n-1}) \cdot \ldots \cdot R(\tau_2) \cdot R(\tau_1))\Big\}$$

$$+ \gamma \Big\{ (F(t_n) \cdot R(\tau_n) \cdot R(\tau_{n-1}) \cdot \ldots \cdot R(\tau_2) \cdot R(\tau_1))\Big\}\Big] \quad ,$$

where the various terms are defined as:

C_{r_1} = possible capacity of on-ramp (vph);

V_s = shoulder-lane volume (vph);

i = index of potential capacity of a given gap (vehicles);

t_i = [illegible] can [illegible] a [illegible]

$P(t_0)$ = [illegible] that fail [illegible] and t_0;

$P(t_i)_{i=1 \text{ to } n}$ = proportion of total [illegible] that fail between t_{i-1} and t_i;

$\bar{T}_i$ = [illegible] that fail between t_{i-1} and t_i;

$P(\bar{T}_i)$ = [illegible] of length $\bar{T}_i$;

$R(\bar{T}_i)$ = [illegible] of length $\bar{T}_i$ [illegible]

n = [illegible]

Of the several [illegible] numerical solution could be [illegible] only one is [illegible] three [illegible] situations [illegible]. This [illegible] $P(t_i)$ the proportion of [illegible] t_{i-1} and t_i [illegible] the shifted-exponential [illegible]. For a given volume the probability that a [illegible] will be [illegible] t_i [illegible] as follows:

$$P(h \geq t_i) = [illegible]$$

where $P(h \geq t_i)$, $\bar{t}$, and $\bar{t}$ are as defined in the previous section.

Of course, this expression can be simplified

$$P(h \geq t_i) = e^{\delta D} \cdot e^{-\delta t_i}$$

$$= k\, e^{-\delta t_i}$$

where:

$$\delta = 1/(\bar{t} - D)$$

and

$$k = e^{\delta D}$$

Since the proportion of gaps that fall into any time range is equal to the difference between the proportion of gaps that are longer than the lower limit and the proportion that are longer than the upper limit,

$$P(t_i) = P(h \geq t_{i-1}) - P(h \geq t_i)$$

or

$$P(t_i) = k\,(e^{-\delta t_{i-1}} - e^{-\delta t_i})\,.$$

The remaining functions in the ramp capacity model, $P(T_i)$ and $E(T_i)$, denote the expected probability with which gaps falling between t_{i-1} and t_i will be accepted and rejected, respectively. Since all of the gaps between any two limits are evaluated in mass, it was necessary to define a representative gap. The mean of the gaps in the range is probably the most appropriate function for this purpose. It is defined by the expression,

$$t_i = \frac{t_{i-1}\,e^{-\delta t_{i-1}} - t_i\,e^{-\delta t_i}}{e^{-\delta t_{i-1}} - e^{-\delta t_i}} + \frac{1}{\delta}\,.$$

Numerical evaluation of $P(T_i)$ was obtained by substituting T_i into the appropriate gap-acceptance equation. $E(T_i)$ was set equal to $(1 - P(T_i))$. The general assumptions for operation at possible capacity are similar for the three design-control situations. As each opening occurs in the shoulder lane a stopped queue is assumed to be on the ramp. The first driver in the queue must make his decision to accept or reject the gap using the appropriate gap-acceptance model. (There was one decision model for a stopped first-in-line vehicle on a ramp with an acceleration lane and a second model for stopped first-in-line vehicles on ramps without acceleration lanes.) When the first vehicle of a ramp queue accepts a gap, he must fall in behind the

shoulder-lane vehicle with a time clearance of 0.5 seconds. Although this is a very short time spacing the leading shoulder-lane vehicle will be traveling at a higher speed in most instances and will consequently increase the clearance. With yield-sign or no-sign control, trailing ramp vehicles enter the shoulder lane at intervals of 2.0 and 1.8 seconds, respectively, provided the remaining log in the shoulder lane is acceptable to them. This acceptance decision is based upon non-stop gap-acceptance models separately defined for the acceleration-lane and no acceleration-lane situations. Of course, with an acceleration lane and stop-sign control every vehicle is required to come to a stop in the first-in-line position before entering the shoulder lane, and as a consequence these drivers use the same gap-acceptance decision model regardless of their position at the beginning of the gap. Vehicles accepting successive positions in the same gap enter the shoulder lane at equal intervals of 4.45 seconds.

The limits over which the possible-capacity queuing model was evaluated for each of the design-control conditions were defined by the appropriate gap-acceptance models and by the established minimum clearance times. These limits are presented in Table 1.

The solution of the possible-capacity queuing model for the three design-control conditions was programmed for the IBM 7090 computer using FORTRAN IV coding.

Monte Carlo Simulation of Practical Capacity

The practical capacity of a freeway on-ramp was defined earlier as the maximum volume of vehicles that can enter the through highway during one hour with 85 percent of the drivers being able to leave the

TABLE 1

TIME LIMITS FOR NUMERICAL SOLUTION OF POSSIBLE-CAPACITY QUEUING MODELS

Index i	t_i--By Design-Control Conditions		
	No Accl-Stop	No Accl-Yield	Accl-No Control
0	3.30	3.30	2.50
1	8.25	5.80	4.80
2	12.70	7.80	6.60
3	17.15	9.80	8.40
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
i	8.25+(i-1)4.45	5.80+(i-1)(2.0)	4.80+(i-1)1.9

ramp without being delayed for more than 60 seconds. Assuming that the flow of ramp traffic into a free way is governed only by the shoulder-lane traffic stress, it was possible to build a relatively simple simulation model for analysis of the situation.

The Simulator Programs

The on-ramp traffic simulators were programmed utilizing both open and closed subroutines under the control of a monitor or master program. The main advantage of this type of structuring is the relative simplicity with which small segments of the overall model can be isolated, programmed, tested, and debugged. In fact, by documenting each of the segmented programs with descriptive comments, written in English, it was possible to prepare completely intelligible simulator programs without first preparing flow diagrams. The advantage of a program that can be readily digested by both the engineer and the computer is obvious.

Sampling the Simulated Traffic

Simulation runs were initiated with an empty system. That is, there were no vehicles in the simulation area when relative simulation time was zero. If the traffic characteristics of the first few simulated vehicles had been recorded and considered in the analysis of the level of performance, they would undoubtedly have biased the results. In order to guard against this bias the simulator was loaded prior to the actuation of the surveillance system. This pre-loading was effected by simulating the flow of 300 ramp vehicles through the ramp area; of course the shoulder-lane flow was simulated simultaneously, but the number of shoulder-lane vehicles involved in the pre-loading operation

was a function of the ratio of shoulder-lane volume to ramp volume. During this initial period no delay or queuing characteristics were recorded. The number of ramp vehicles that were simulated for pre-loading purposes was established arbitrarily; but it was assumed that 300 vehicles (an average of approximately one-half hour of real traffic flow) was adequate to establish equilibrium conditions in the ramp area.

Following the pre-loading operation the surveillance system was actuated and an additional 1000 ramp vehicles were generated and observed. In this case the sample size was established by a dollars constraint rather than by statistical design. After estimates of running time had been prepared from the results of a pilot study, sample sizes were established to conform with the available project funds.

Descriptors of Traffic Performance

The level of performance in the ramp system is defined for every combination of ramp and shoulder-lane volumes and for each design-control situation by six variables. Listed under the headings of queuing characteristics and delay characteristics these variables are as follows:

A. Queuing Characteristics

1. average queue length
2. 85th percentile queue length
3. 90th percentile queue length
4. 95th percentile queue length

B. Delay Characteristics

5. average delay
6. probability that delay exceeds 60 seconds.

Computer Programs

The practical-capacity simulators for the three design-control combinations were programmed to be processed on the IBM 7090 Computer using FORTRAN IV and MAP coding (5).

RESULTS AND DISCUSSION

The results from the queuing and simulation analyses are presented separately. Solution of the queuing models led directly to the definitions of numerical, possible capacity limits for the three freeway on-ramp design-control conditions--no acceleration lane with stop-sign control, no acceleration lane with yield-sign control, and an acceleration lane with no sign control. These numerical definitions are presented in graphical form. In contrast, the results obtained from the simulation did not directly define practical capacities. Each simulation run produced a record describing delay and queue characteristics at various combinations of shoulder-lane and ramp volumes. Subsequent statistical analyses of the delay characteristics provided the basis for the definitions of practical capacity. Related queuing characteristics are described by both graphical and mathematical models,

Results of the Queuing Analysis

Numerical Limits for Possible Capacity

The possible capacities of freeway on-ramps obtained from solutions of the queuing model are shown in Figure 4 for the three design-control combinations. Although there was practically no scatter in the possible capacity plots, statistical analyses were made for the purpose of developing empirical prediction models. Nearly perfect least-square fits were obtained using an equation of the form,

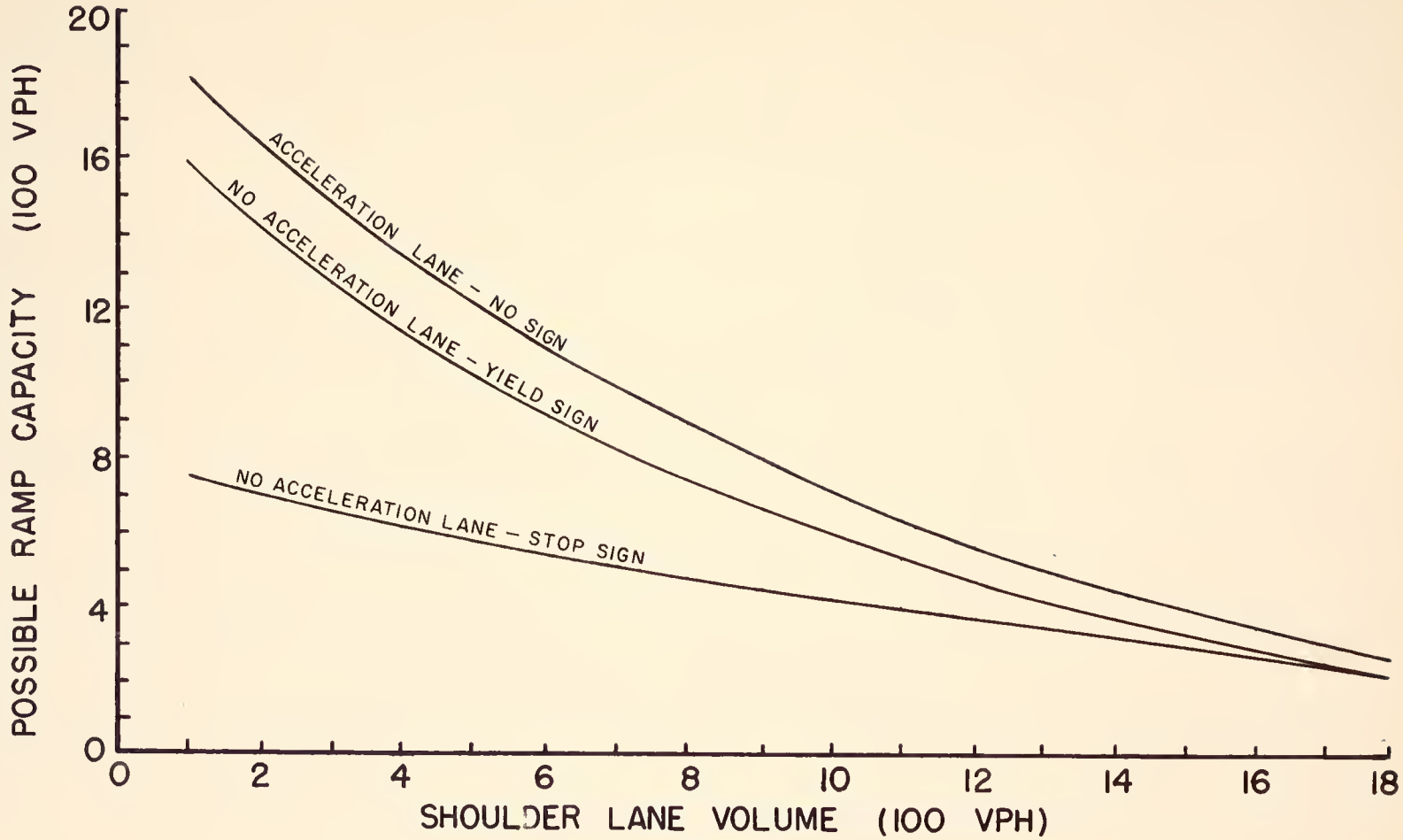


FIG. 4 - POSSIBLE CAPACITIES OF FREEWAY ON-RAMPS

TABLE 2

SUMMARY OF LEAST-SQUARE EQUATIONS FOR PREDICTING POSSIBLE CAPACITIES OF FREEWAY ON-RAMPS

Prediction Model: $y = e^{(a+bx+cx^2)}$*

Regression Coefficients			Statistical Characteristics			
			Limits of Analysis-x			
a	b	c	Low	High	R^2	Number of Observations
No Acceleration Lane and Stop-Sign Control						
+6.6478	-0.0004829	-0.0000001038	100	1800	.999	18
No Acceleration Lane and Yield-Sign Control						
+7.4567	-0.0009773	-0.0000000813	100	1800	.999	18
Acceleration Lane and No Sign Control						
+7.5888	-0.0008983	-0.0000001099	100	1800	.999	18

* y = Possible Capacity -- vph
x = Shoulder-Lane Volume -- vph

$$y = e^{(a + bx + cx^2)} .$$

The results of these statistical analyses are summarized in Table 2 for the three design-control combinations. As expected the on-ramp with an acceleration lane and no sign control had the highest possible capacity; the lowest capacity was realized on the on-ramp with no acceleration lane and stop-sign control. The capacity of the ramp with no acceleration lane and yield-sign control approaches that of the ramp with an acceleration lane and no sign control at low shoulder-lane volumes. This is readily explained. At shoulder-lane volumes approaching zero flow the ramp stream can move almost continuously with minimum time spacings between successive vehicles. Since there is only a small difference in the minimum allowable spacings for the two conditions under consideration the potential capacities approach each other, although they cannot be equal. With no acceleration lane and yield-sign control the 2.0 seconds minimum spacing permits a maximum capacity potential of 1800 vehicles per hour; a maximum capacity potential of 2000 vehicles per hour is possible with the 1.8 second minimum time spacing imposed on ramp vehicles with an acceleration lane available. Little or no capacity difference results from differences in the gap-acceptance models for the two situations. At volumes approaching zero in the shoulder lane almost all gaps are long enough to be completely acceptable to all ramp drivers.

As shoulder-lane volumes increase from zero, however, the length of available gaps decreases with the consequence that more of the vehicles on the ramp with yield-sign control are forced to stop than on the ramp with an acceleration lane. Since gap-acceptance differences become even more critical after a stop, the possible capacities of the two ramp situations become widely divergent.

At high shoulder-lane volumes the possible capacity of the yield-sign controlled ramp approaches that of the stop-sign controlled ramp. This can be attributed to the similarity between stop- and yield-sign control that occurs as traffic conditions on the shoulder lane become congested. With stop-sign control, a stop is mandatory before entering the shoulder lane; with yield-sign control there is no absolute stop requirement, but due to the shortage of acceptable gaps in the shoulder-lane most ramp vehicles find it necessary to stop before merging with the shoulder-lane stream. Of course all vehicles stopped in the first-in-line position on a ramp without an acceleration lane must utilize the same decision model for gap acceptance regardless of the type of control.

Simulation Results

Generated Versus Requested Volumes

The ramp and shoulder-lane traffic flows were generated by a simulated-sampling technique whereby theoretical headway distributions were sampled using random numbers. At the start of each simulation run parameters of the headway distributions were established for the particular volumes desired. The generated volumes varied from the requested volumes due partly to sampling error, and partly to small errors inherent in the equations for predicting the volume related distribution parameters.

The requested and generated ramp volumes are compared in Table 3. In ten of the twelve volume conditions considered the simulated ramp volumes were slightly lower than the requested volumes. Since each simulation run was continued until 1300 ramp vehicles had been generated,

TABLE 3

COMPARISON OF RAMP VOLUMES GENERATED BY SIMULATOR WITH RAMP VOLUMES REQUESTED

Ramp Volumes -- (VPH)

Requested	Generated
100	92
200	194
300	297
400	405
500	503
600	598
700	694
800	783
900	872
1000	957
1100	1052
1200	1156

and since the same sequence of random numbers was utilized for each run, identical ramp volumes were generated each time the same volume was requested. In contrast the number of shoulder-lane vehicles generated at a given volume level was dependent upon the ramp volume being generated with the result that the simulated shoulder-lane volumes were different for almost every run. A comparison between the requested and the generated shoulder-lane volume is given in Table 4. Again it should be noted that the majority of the generated volumes are slightly lower than the requested volumes.

Traffic Performance By Type of On-Ramp

Traffic performance for each of the three different ramp designs was described for every combination of ramp and shoulder-lane volumes by six variables--the average length of queue found on the ramp, the 85th, 90th, and 95th percentile queue lengths found on the ramp, the average delay incurred by a ramp vehicle, and the probability that the delay incurred by a ramp vehicle exceeds 60 seconds. Statistical analyses were performed on the data describing the average queue length, the average delay, and the probability that delay exceeds 60 seconds; and least-quare prediction models were constructed to explain the variation in each of these characteristics as a function of shoulder-lane volume, with ramp volume held constant at each of several different level.

No Acceleration Lane--Stop-Sign Control. Analyses of the stop-sign controlled ramp situation were conducted for shoulder-lane volumes ranging from 100 to 1800 vehicles per hour. The range of ramp volumes studied at each shoulder-lane volume varied from 100 vehicles per hour to the possible capacity of the ramp defined for the given shoulder-

TABLE 4

SHOULDER-LANE VOLUMES GENERATED BY SIMULATOR AT
VARIOUS REQUESTED COMBINATIONS OF RAMP AND SHOULDER-LANE TRAFFIC VOLUMES

Shoulder-Lane Volume Requested -- (VPH)	Ramp Volume Requested -- (VPH)											
	100	200	300	400	500	600	700	800	900	1000	1100	1200
100	100	97	93	91	89	89	90	91	85	82	80	77
200	197	200	194	194	185	186	182	180	180	178	180	178
300	300	297	302	294	291	291	282	280	277	277	271	267
400	395	396	394	402	393	387	391	390	381	373	373	371
500	498	494	494	497	503	493	493	482	489	487	481	471
600	598	598	595	594	598	603	587	590	584	576	587	585
700	694	701	694	693	694	693	704	690	687	689	677	672
800	792	792	791	791	788	789	799	804	795	790	784	788
900	891	890	897	887	893	893	891	900	900	903	895	
1000	992	993	1003	990	991	989	994	990	993	994		
1100	1097	1098	1099	1089	1086	1095	1084	1092	1089			
1200	1199	1198	1190	1200	1188	1191	1192	1193				
1300	1298	1296	1287	1300	1282	1283						
1400	1393	1390	1386	1402	1396							
1500	1495	1490	1494	1498	1502							
1600	1596	1587	1593	1585								
1700	1695	1685	1698									
1800	1796	1783										

lane volume. Since the maximum ramp capacity that can be realized even at very low shoulder-lane volumes is only slightly in excess of 700 vehicles per hour no simulation runs were considered above this limit. The results describing average queue length, average delay, and the probability that delay exceeds 60 seconds are plotted in Figures 5, 6, and 7, respectively. In each case the plots represent empirical equations fitted to observed data by the "method of least-squares." A separate analysis was made to determine the relationship between a given characteristic (average queue, average delay, etc.) and shoulder-lane volume at each level of ramp volume.

The results of the statistical analysis of each family of curves are summarized in the table immediately following each figure. The equations fitted to the three characteristics were all of the same general form,

$$y = e^{(a + bx + cx^2)},$$

where:

y = the ramp characteristic under consideration, and

x = shoulder-lane volume expressed in vehicles per hour.

The regression coefficients, multiple R^2's, ranges of analyses, and the number of observations included in each analysis are presented in Tables 5, 6, and 7. The apparent amounts of variability (R^2) explained by each of the derived equations are very high. In 11 out of 21 fits the R^2's had values in excess of 0.980, and in 17 of the 21 fits they exceeded 0.950. In no case was an R^2 of less than 0.936 obtained. Care should be exercised, however, in interpreting the significance of these values. They are not true estimates of the proportion of the variability in the y's that is explained by the model; rather, they

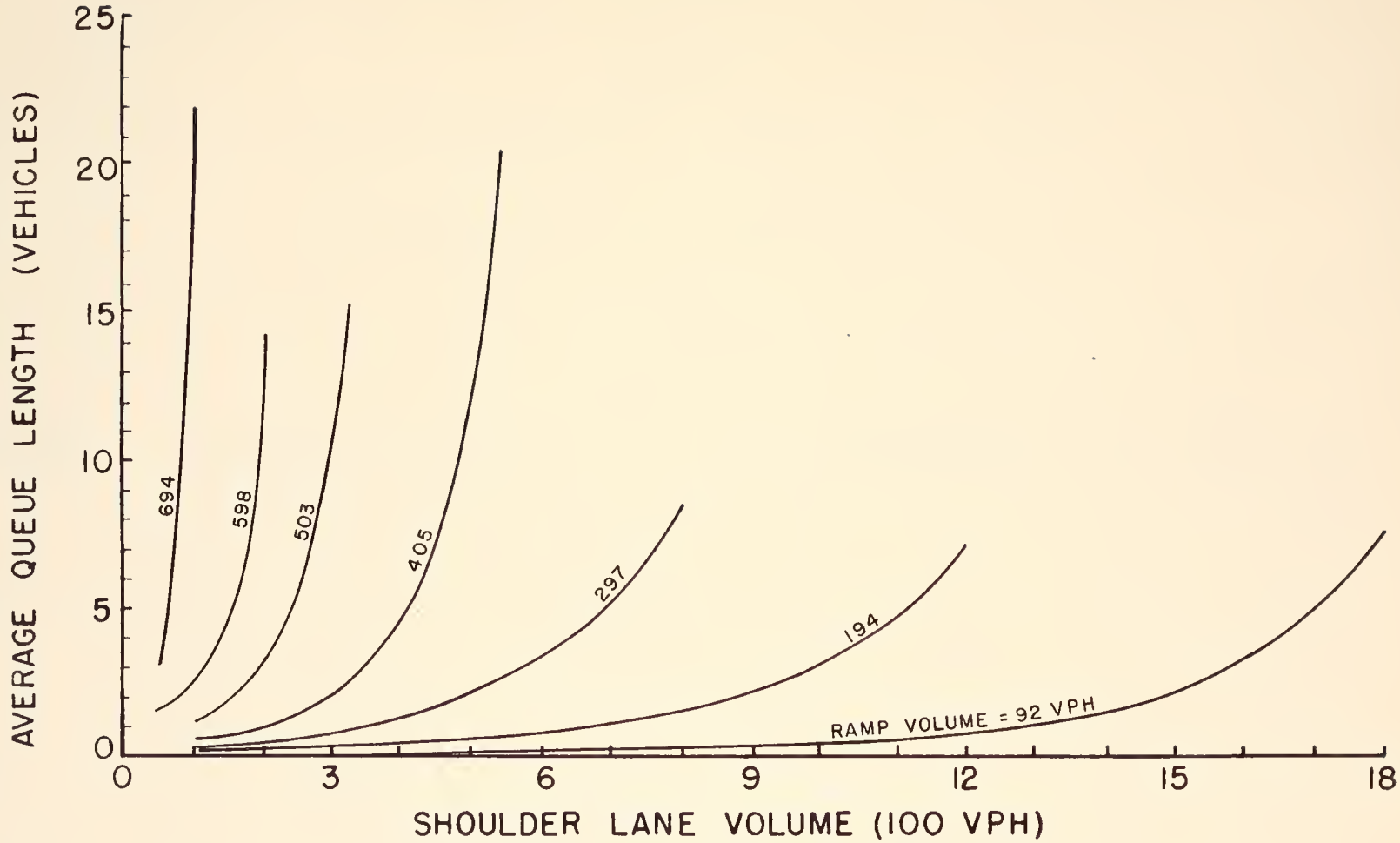


FIG. 5 - AVERAGE QUEUE LENGTH ON RAMP WITH STOP SIGN CONTROL

TABLE 5

EQUATIONS FOR PREDICTION OF AVERAGE QUEUE LENGTHS

No Acceleration Lane -- Stop-Sign Control

Prediction Model: $y = e^{(a+bx+cx^2)}$ where x is shoulder-lane volume expressed in vehicles per hour.

Ramp Volume (VPH)	Regression Coefficients			Statistical Characteristics			
				Range of Analysis			Number of
	a	b	c	Low x	High x	R^2	Observations
100	-1.9975	+.0000625	+.0000012797	100	1800	.963	18
200	-1.6397	+.0018591	+.0000009418	100	1200	.971	12
300	-1.5500	+.0043815	+.0000002995	100	800	.984	8
400	-0.9395	+.0032663	+.0000071594	100	600	.996	6
500	-0.7899	+.0082321	+.0000079973	100	400	.999	7
600	+0.3085	-.0010991	+.0000643069	50	225	.998	8
700	-2.0566	+.0751588	-.0002365566	50	150	.972	5

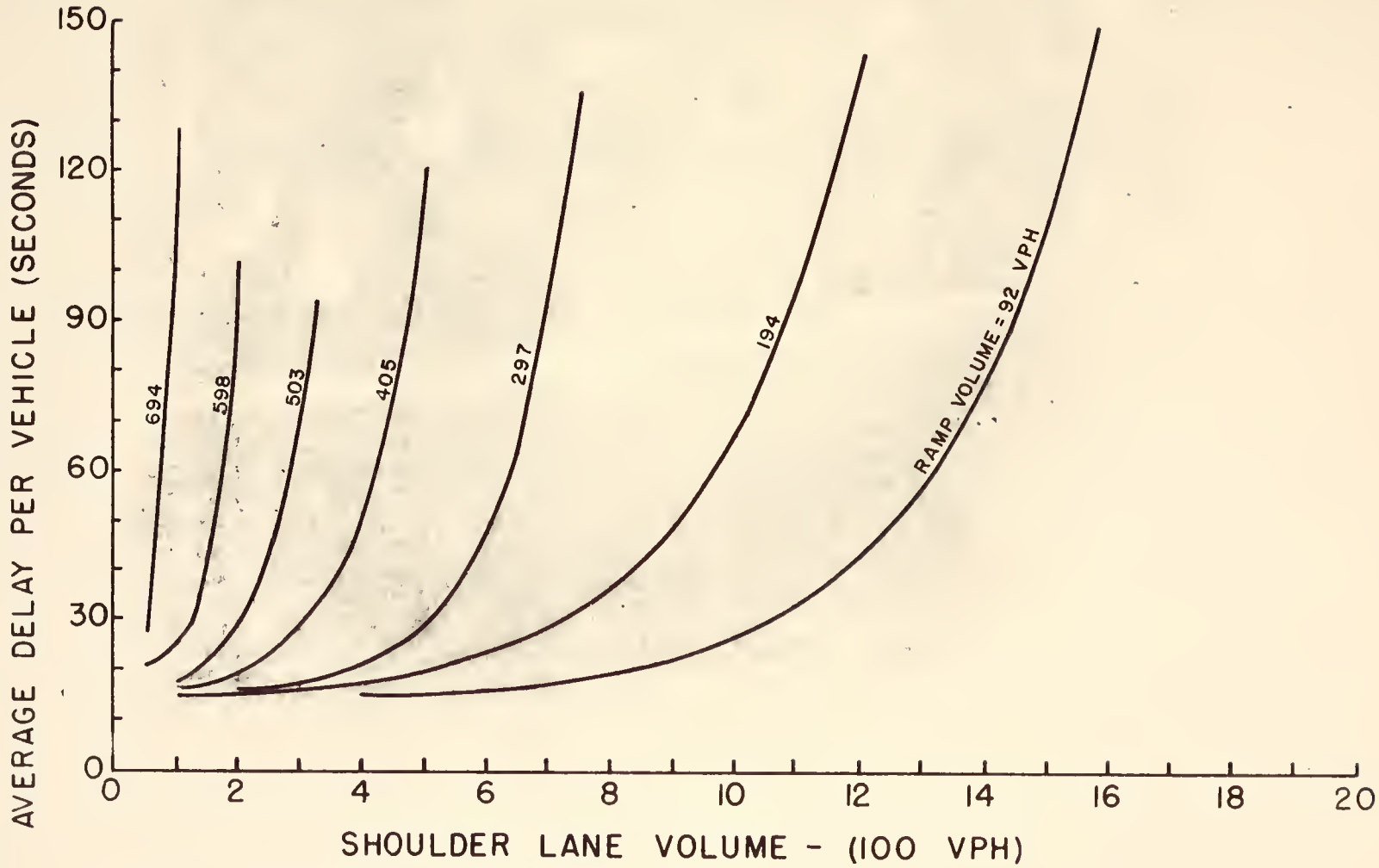


FIG. 6 - AVERAGE DELAY TO RAMP VEHICLES WITH STOP SIGN CONTROL

TABLE 6

EQUATIONS FOR PREDICTION OF AVERAGE DELAY -- (SECONDS)

No Acceleration Lane -- Stop-Sign Control

Prediction Model: $y = e^{(a+bx+cx^2)}$ where x is shoulder-lane volume expressed in vehicles per hour.

Ramp Volume (VPH)	Regression Coefficients			Statistical Characteristics			
				Range of Analysis			Number of
	a	b	c	Low x	High x	R^2	Observations
100	+2.9952	-.0013762	+.0000016831	100	1800	.938	18
200	+2.7652	-.0004893	+.0000019405	100	1200	.967	12
300	+3.1612	-.0034973	+.0000077949	100	900	.947	9
400	+2.8651	-.0019709	+.0000116908	100	600	.998	6
500	+2.6811	-.0004187	+.0000189636	100	400	.999	7
600	+3.2122	-.0072780	+.0000717930	50	225	.996	8
700	+1.0967	+.0513798	-.0001375146	50	150	.979	5

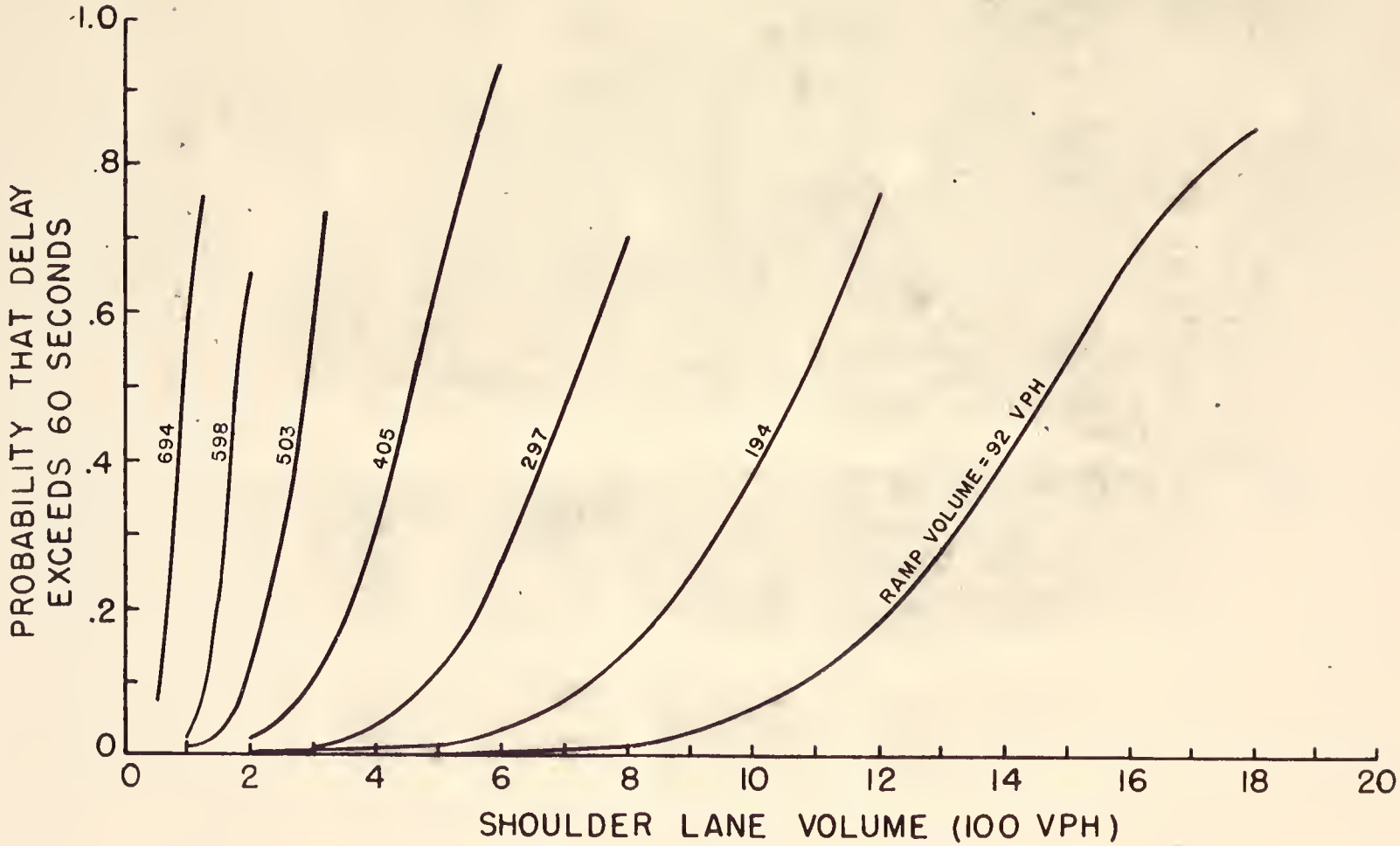


FIG. 7 - PROBABILITY THAT DELAY EXCEEDS 60 SECONDS WITH STOP SIGN CONTROL

TABLE 7

EQUATIONS FOR PREDICTION OF

PROBABILITY THAT DELAY IS GREATER THAN 60 SECONDS

No Acceleration Lane -- Stop-Sign Control

Prediction Model: $y = e^{(a+bx+cx^2)}$ where x is shoulder-lane volume expressed in vehicles per hour.

Ramp Volume (VPH)	Regression Coefficients			Statistical Characteristics			
				Range of Analysis			Number of
	a	b	c	Low x	High x	R^2	Observations
100	-11.7913	+.0123782	-.0000032853	500	1800	.985	14
200	-9.3662	+.0126359	-.0000042133	200	1200	.936	11
300	-9.5943	+.0205038	-.0000111879	300	800	.969	6
400	-8.0900	+.0249755	-.0000193204	200	600	.996	5
500	-9.4459	+.0493143	-.0000651674	100	400	.995	7
600	-13.3415	+.1252529	-.0003035478	100	225	.985	7
700	-6.6710	+.1034048	-.0004176055	50	120	.941	5

are estimates of the proportion of the variability in the natural logarithms of the y's that are explained by the transformed model,

$$\ln(y) = a + bx + cx^2.$$

No Acceleration Lane--Yield-Sign Control. No ramp volumes were studied in excess of 1200 vehicles per hour although the maximum possible capacity of a ramp with yield-sign control is nearly 1600 vehicles per hour under low shoulder-lane volume conditions. Analysis was restricted to this lower limit due to the characteristics of the ramp-vehicle generator. The parameters of the ramp headway distribution have only been defined for volumes in the range of 100 to 1200 vehicles per hour.

Graphical representations of the empirical models describing average queue length, average delay, and the probability that a vehicle incurs delay in excess of 60 seconds are shown in Figures 8, 9, and 10, respectively. Summaries of the results of the statistical analyses performed to obtain least square equations for each of these characteristics are given in Tables 8, 9, and 10. Similar to the stop-sign analyses, multiple R^2's in excess of .964 were obtained for fits to the natural log transformation of an equation of the form,

$$y = e^{(a + bx + cx^2)}.$$

Acceleration Lane--No Sign Control. Although a maximum possible capacity of approximately 1800 vehicles per hour can be realized on a ramp with an acceleration lane and no sign control, provided the shoulder-lane volume is very low, the range of analysis was again restricted to a maximum ramp volume of 1200 vehicles because of the limitations on the ramp-vehicle generator.

The graphical representations of the empirical models describing

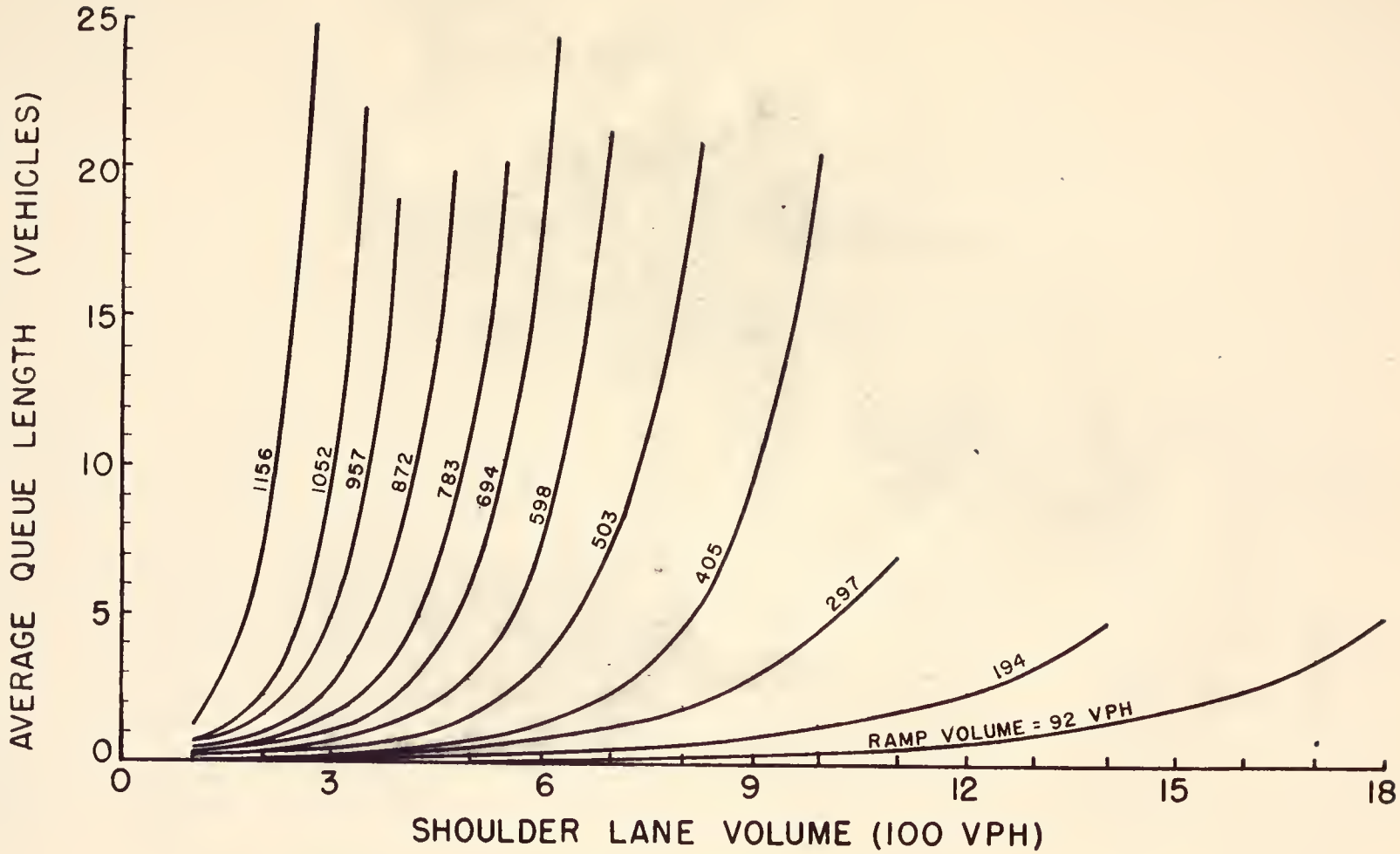


FIG. 8 - AVERAGE QUEUE LENGTH ON RAMP WITH YIELD SIGN CONTROL

TABLE 8

EQUATIONS FOR PREDICTION OF AVERAGE QUEUE LENGTHS

No Acceleration Lane -- Yield-Sign Control

Prediction Model: $y = e^{(a+bx+cx^2)}$ where x is shoulder-lane volume expressed in vehicles per hour.

Ramp Volume (VPH)	Regression Coefficients			Statistical Characteristics			
				Range of Analysis			Number of
	a	b	c	Low x	High x	R^2	Observations
100	-4.2729	+.0033557	-.0000000456	100	1800	.977	18
200	-3.2649	+.0035794	-.0000001103	100	1400	.964	14
300	-3.0580	+.0048794	-.0000002977	100	1100	.973	11
400	-2.5712	+.0032686	-.0000023291	100	1000	.982	10
500	-2.4593	+.0045387	-.0000025817	100	900	.972	9
600	-1.6940	+.0021831	-.0000065882	100	800	.988	8
700	-1.5052	+.0026259	-.0000078505	100	700	.987	7
800	-1.4855	+.0039350	-.0000076947	100	700	.996	7
900	-1.2510	+.0042842	-.0000097527	100	600	.994	6
1000	-1.1467	+.0055615	-.0000116985	100	500	.991	5
1100	-1.4023	+.0081140	-.0000135458	100	400	.999	4
1200	-2.2924	+.0261991	-.0000222119	100	450	.994	8

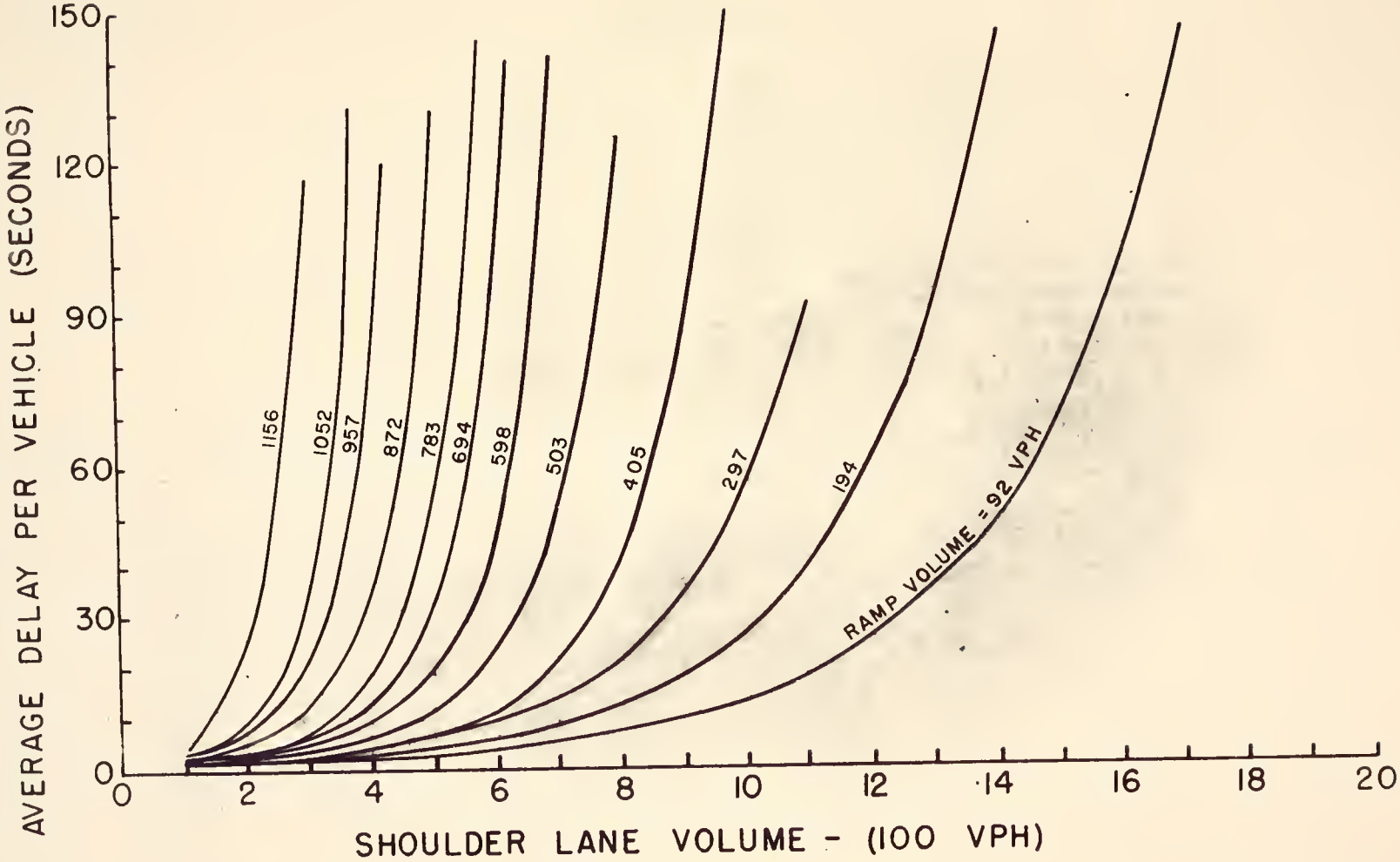


FIG. 9 - AVERAGE DELAY TO RAMP VEHICLES WITH YIELD SIGN CONTROL

TABLE 9

EQUATIONS FOR PREDICTION OF AVERAGE DELAY -- (SECONDS)

No Acceleration Lane -- Yield-Sign Control

Prediction Model: $y = e^{(a+bx+cx^2)}$ where x is shoulder-lane volume expressed in vehicles per hour.

Ramp Volume (VPH)	Regression Coefficients			Statistical Characteristics			
				Range of Analysis			Number of
	a	b	c	Low x	High x	R^2	Observations
100	-0.0969	+.0022760	+.0000004194	100	1800	.984	18
200	+0.0931	+.0025321	+.0000006884	100	1400	.973	14
300	-0.1598	+.0036479	+.0000005528	100	1100	.984	11
400	+0.0532	+.0024242	+.0000027514	100	1000	.981	10
500	+0.1778	+.0024766	+.0000041782	100	900	.975	9
600	+0.5665	+.0009040	+.0000076655	100	800	.988	8
700	+0.6343	+.0010002	+.0000094279	100	700	.986	7
800	+0.5380	+.0019045	+.0000101179	100	600	.990	6
900	+0.6158	+.0028395	+.0000113563	100	600	.996	6
1000	+0.5615	+.0041011	+.0000137510	100	500	.994	5
1100	+0.4076	+.0056497	+.0000167166	100	400	.999	4
1200	-0.2721	+.0189543	-.0000071903	100	400	.994	7

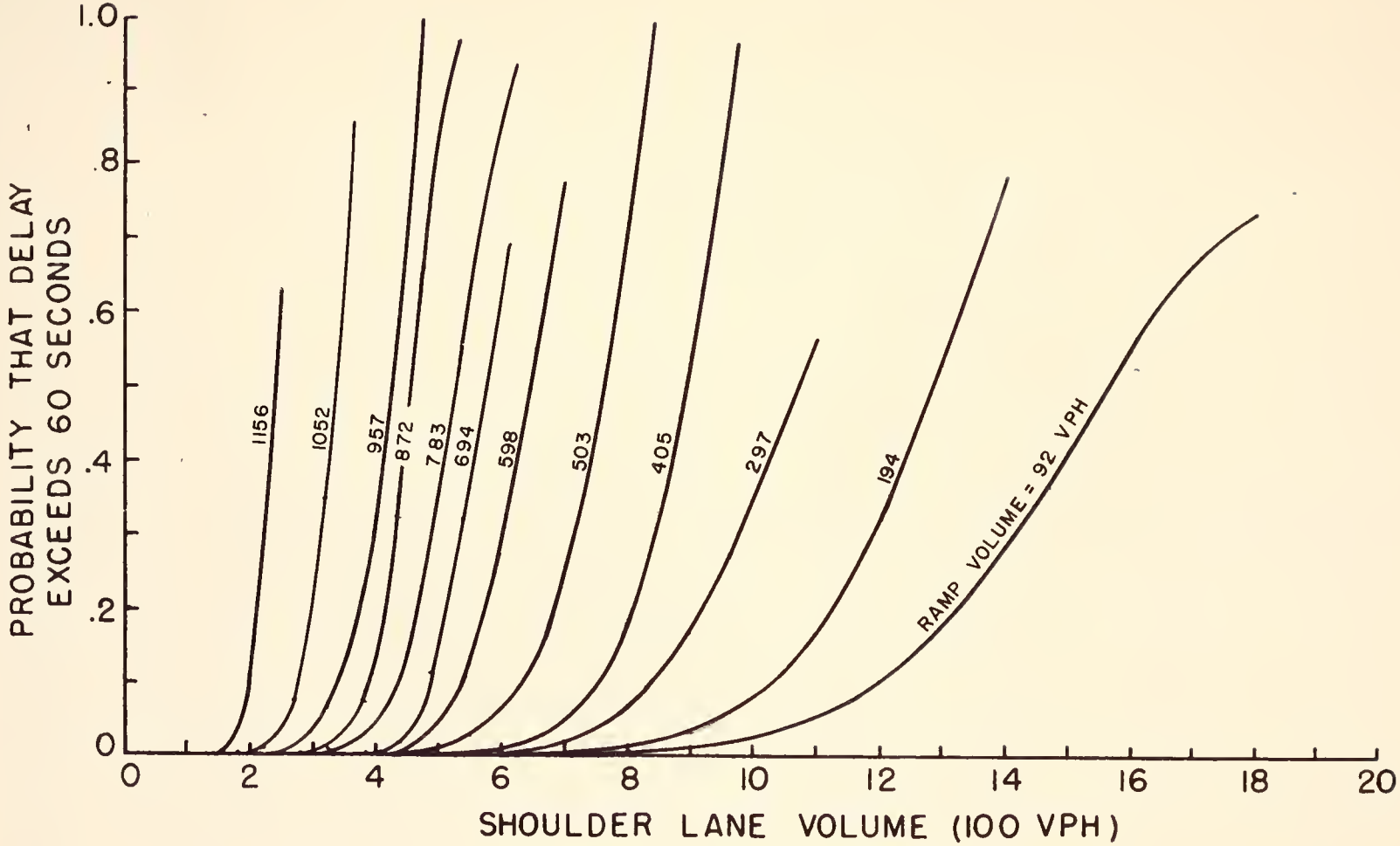


FIG. 10 - PROBABILITY THAT DELAY EXCEEDS 60 SECONDS WITH YIELD SIGN CONTROL

TABLE 10

EQUATIONS FOR PREDICTION OF

PROBABILITY THAT DELAY IS GREATER THAN 60 SECONDS

No Acceleration Lane -- Yield-Sign Control

Prediction Model: $y = e^{(a+bx+cx^2)}$ where x is shoulder-lane volume expressed in vehicles per hour.

Ramp Volume (VPH)	Regression Coefficients			Statistical Characteristics			
				Range of Analysis			Number of
	a	b	c	Low x	High x	R^2	Observations
100	-16.1550	+.0170741	-.0000045908	700	1800	.988	12
200	-15.9561	+.0191254	-.0000056410	600	1400	.988	9
300	-17.5748	+.0271073	-.0000105851	500	1100	.988	7
400	-21.3262	+.0370901	+.0000156363	500	1000	.987	6
500	-19.3923	+.0393628	+.0000194069	400	850	.998	6
600	-29.0753	+.0781986	+.0000528842	400	750	.986	5
700	-47.3614	+.1526573	-.0001346453	400	650	.981	6
800	-21.6138	+.0675914	+.0000529782	300	625	.997	5
900	-28.8463	+.1063333	-.0000980491	300	550	.999	6
1000	-14.7238	+.0494355	+.0000386172	200	500	.966	4
1100	-21.5913	+.1035501	-.0001235440	200	400	.999	5
1200	-26.5800	+.1917022	-.0003487793	150	300	.999	4

average queue length, average delay, and the probability that a vehicle incurs delay in excess of 60 seconds are presented in Figures 11, 12, and 13, respectively. Summaries of the statistical analyses, made in order to obtain least-square estimates of the parameters of the empirical models describing these characteristics, are shown in Tables 11, 12, and 13. All of the relationships were fitted to the same model used previously for the stop-sign and yield-sign controlled ramps; and again, the amounts of variability (R^2) in the logarithms of the dependent variables that were described by the transformed models were in excess of 0.950 in all cases.

Practical Capacity Analysis

Numerical Limits for Practical Capacity

The empirical models describing the probability that a vehicle will incur delay in excess of 60 seconds were utilized to define the practical capacities of the three ramp designs. These models were solved at each level of ramp volume to establish the shoulder-lane volume at which the probability of delay in excess of 60 seconds was 0.15. The resulting ramp volume, shoulder-lane volume data sets described the relationship between practical ramp capacity and shoulder-lane volume. These data sets are plotted for each of the three ramp designs in Figure 14. The curves drawn through these points are least-square fits to a model of the form. The complete analyses are summarized in Table 14.

Queuing Conditions at Practical Capacity

The average queue length models were solved and percentile-queue data were evaluated at practical-capacity volume conditions.

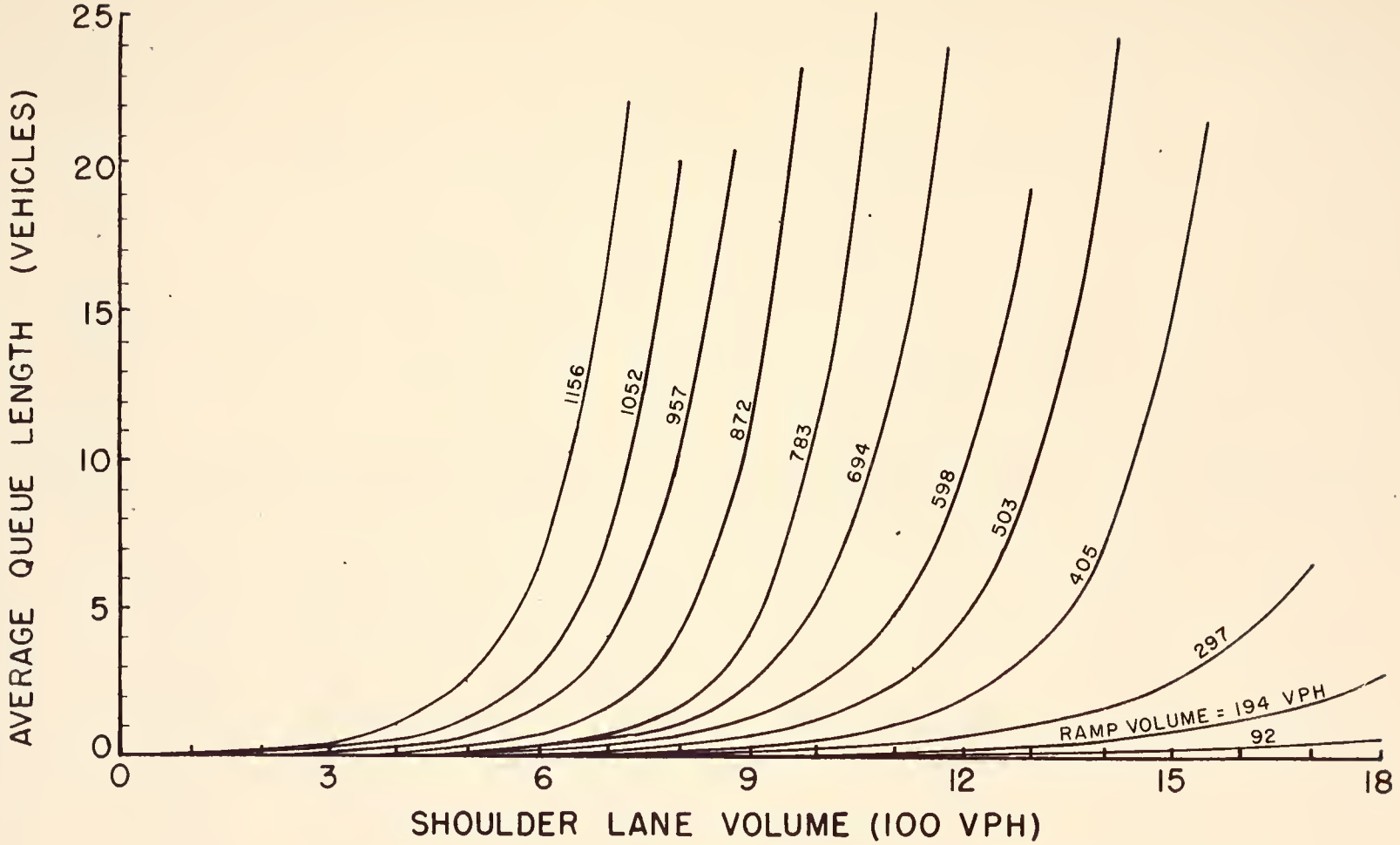


FIG. 11 - AVERAGE QUEUE LENGTH ON RAMP WITH ACCELERATION LANE

TABLE 11

EQUATIONS FOR PREDICTION OF AVERAGE QUEUE LENGTHS

Acceleration Lane -- No Sign Control

Prediction Model: $y = e^{(a+bx+cx^2)}$ where x is shoulder-lane volume expressed in vehicles per hour.

Ramp Volume (VPH)	Regression Coefficients			Statistical Characteristics			
				Range of Analysis			Number of
	a	b	c	Low x	High x	R^2	Observations
100	-5.0258	+.0016410	+.0000005153	200	1800	.978	17
200	-4.6655	+.0022476	+.0000005141	200	1800	.978	17
300	-4.4841	+.0028177	+.0000005433	100	1700	.988	17
400	-4.3162	+.0019374	+.0000018261	100	1600	.974	16
500	-4.2762	+.0027761	+.0000017289	100	1500	.981	15
600	-4.4491	+.0043213	+.0000010611	100	1300	.983	13
700	-3.8462	+.0027600	+.0000027415	100	1200	.989	12
800	-3.0714	+.0004599	+.0000050207	100	1200	.994	12
900	-4.1394	+.0047775	+.0000027603	100	1100	.988	11
1000	-3.5794	+.0053980	+.0000024726	100	1000	.988	10
1100	-3.1336	+.0051631	+.0000031242	100	900	.991	9
1200	-3.0124	+.0065285	+.0000026169	100	800	.984	8

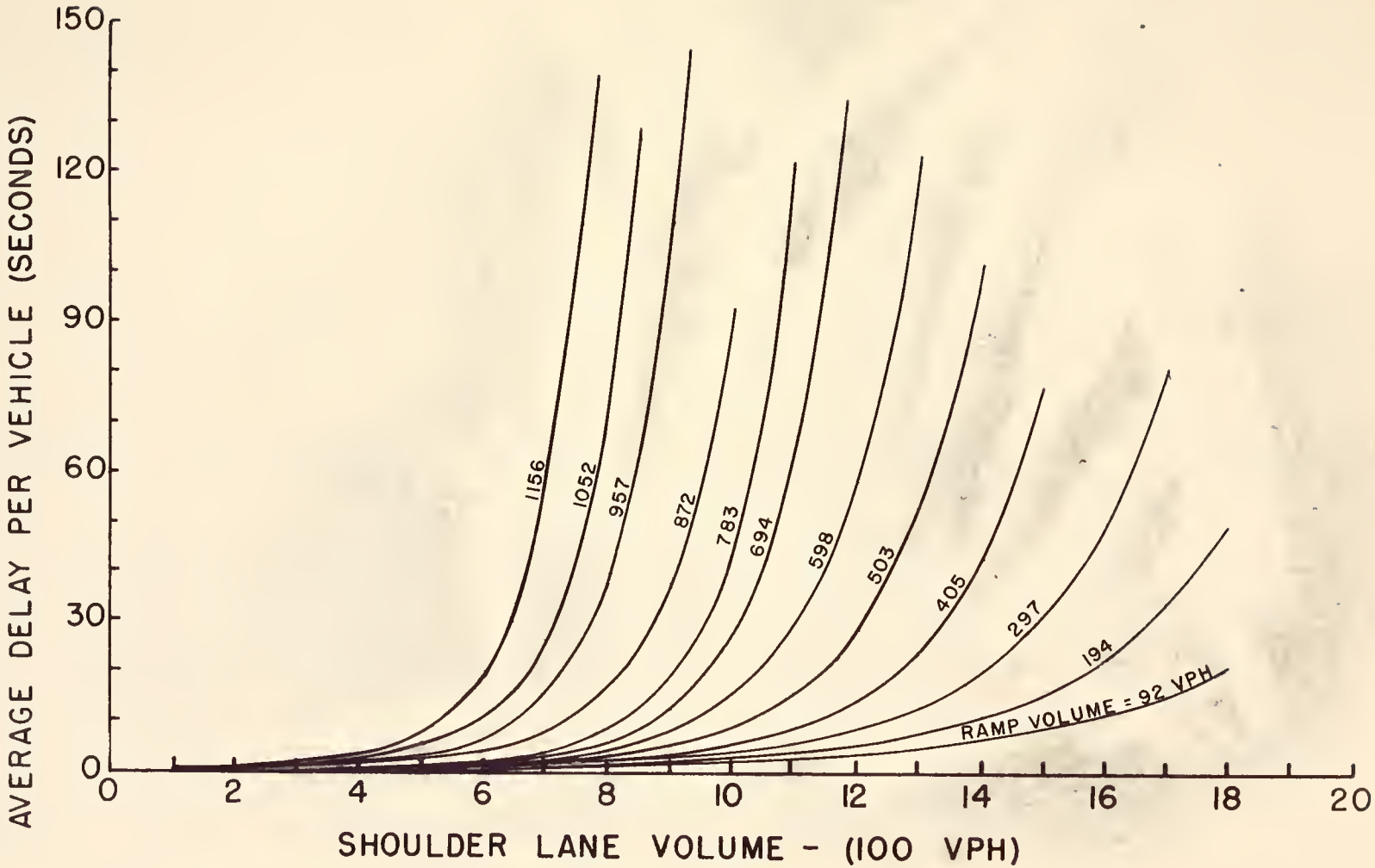


FIG. 12 - AVERAGE DELAY TO RAMP VEHICLES WITH ACCELERATION LANE

TABLE 12

EQUATIONS FOR PREDICTION OF AVERAGE DELAY -- (SECONDS)

Acceleration Lane -- No Sign Control

Prediction Model: $y = e^{(a+bx+cx^2)}$ where x is shoulder-lane volume expressed in vehicles per hour.

Ramp Volume (VPH)	Regression Coefficients			Statistical Characteristics			
				Range of Analysis			Number of
	a	b	c	Low x	High x	R^2	Observations
100	-2.1045	+.0029398	-.0000000386	100	1800	.985	18
200	-1.6750	+.0023434	+.0000004250	100	1800	.990	18
300	-1.6785	+.0023707	+.0000007087	100	1700	.989	17
400	-1.4401	+.0016767	+.0000014566	100	1500	.987	15
500	-1.5498	+.0021741	+.0000015970	100	1400	.987	14
600	-1.4659	+.0019993	+.0000021832	100	1300	.985	13
700	-1.1070	+.0005302	+.0000039077	100	1200	.991	12
800	-0.9499	+.0003271	+.0000044638	100	1100	.995	11
900	-1.4987	+.0027609	+.0000032714	100	1000	.989	10
1000	-1.0421	+.0018696	+.0000050159	100	1000	.992	10
1100	-0.9161	+.0021628	+.0000054528	100	900	.993	9
1200	-0.7589	+.0024525	+.0000063174	100	800	.991	8

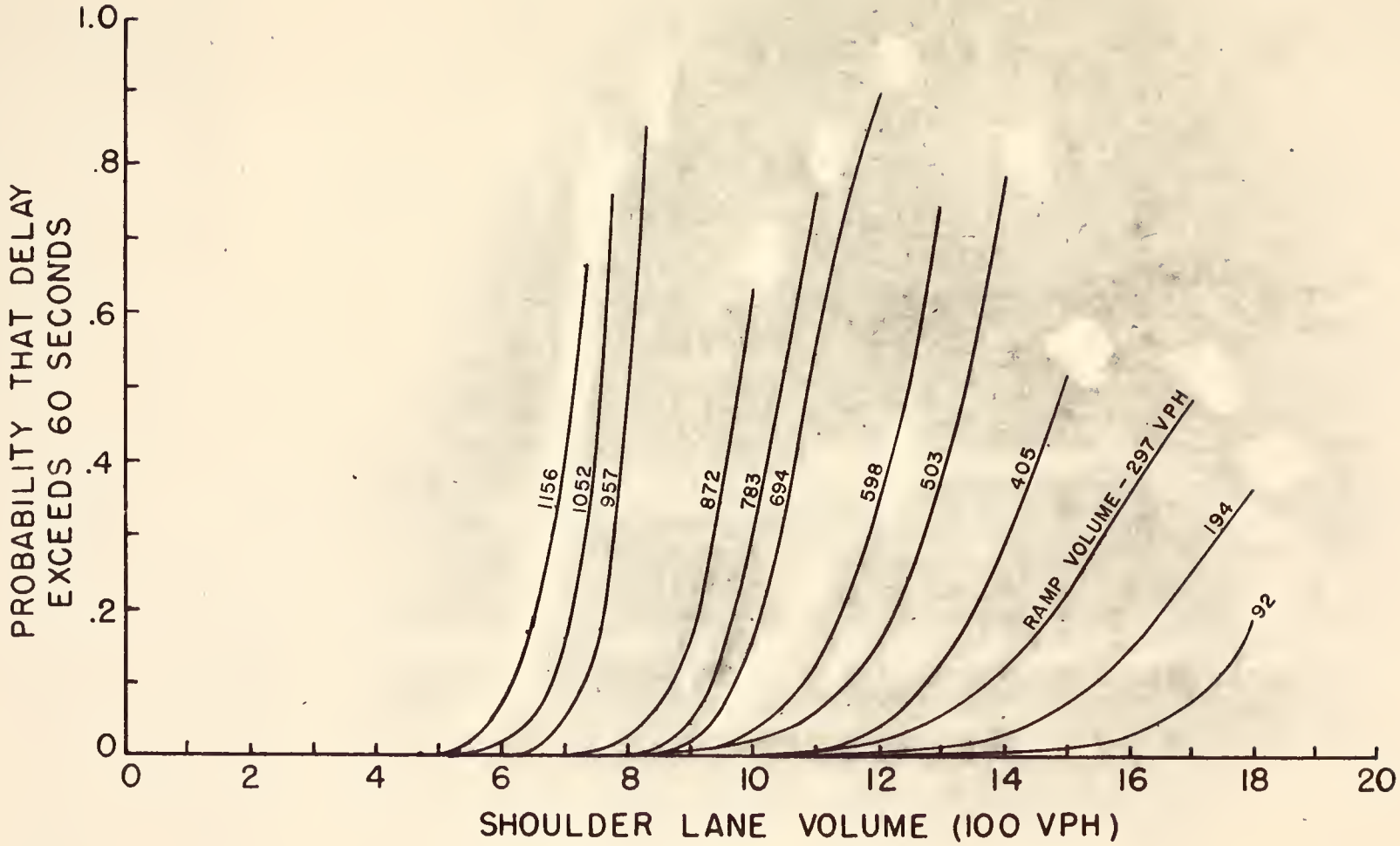


FIG. 13 - PROBABILITY THAT DELAY EXCEEDS 60 SECONDS WITH ACCELERATION LANE

TABLE 13

EQUATIONS FOR PREDICTION OF

PROBABILITY THAT DELAY IS GREATER THAN 60 SECONDS

Acceleration Lane -- No Sign Control

Prediction Model: $y = e^{(a+bx+cx^2)}$ where x is shoulder-lane volume expressed in vehicles per hour.

Ramp Volume (VPH)	Regression Coefficients			Statistical Characteristics			
				Range of Analysis			Number of
	a	b	c	Low x	High x	R^2	Observations
100	-12.7287	+.0026419	+.0000019410	1200	1800	.927	7
200	-32.2409	+.0318443	-.0000080515	1100	1800	.997	8
300	-27.9335	+.0298356	-.0000081325	1000	1700	.937	8
400	-38.0136	+.0454645	-.0000137083	1000	1500	.953	6
500	-19.3308	+.0201150	-.0000046270	800	1400	.944	7
600	-31.4166	+.0417297	-.0000136855	800	1200	.995	5
700	-56.4630	+.0927711	-.0000381728	800	1200	.989	5
800	-59.8336	+.1030377	-.0000444457	800	1100	.991	4
900	-42.7534	+.0738793	-.0000315796	700	1000	.999	4
1000	-39.8221	+.0778939	-.0000361358	600	850	.995	5
1100	-21.1578	+.0339636	-.0000090505	500	800	.999	5
1200	-41.9321	+.1061389	-.0000674651	500	750	.999	6

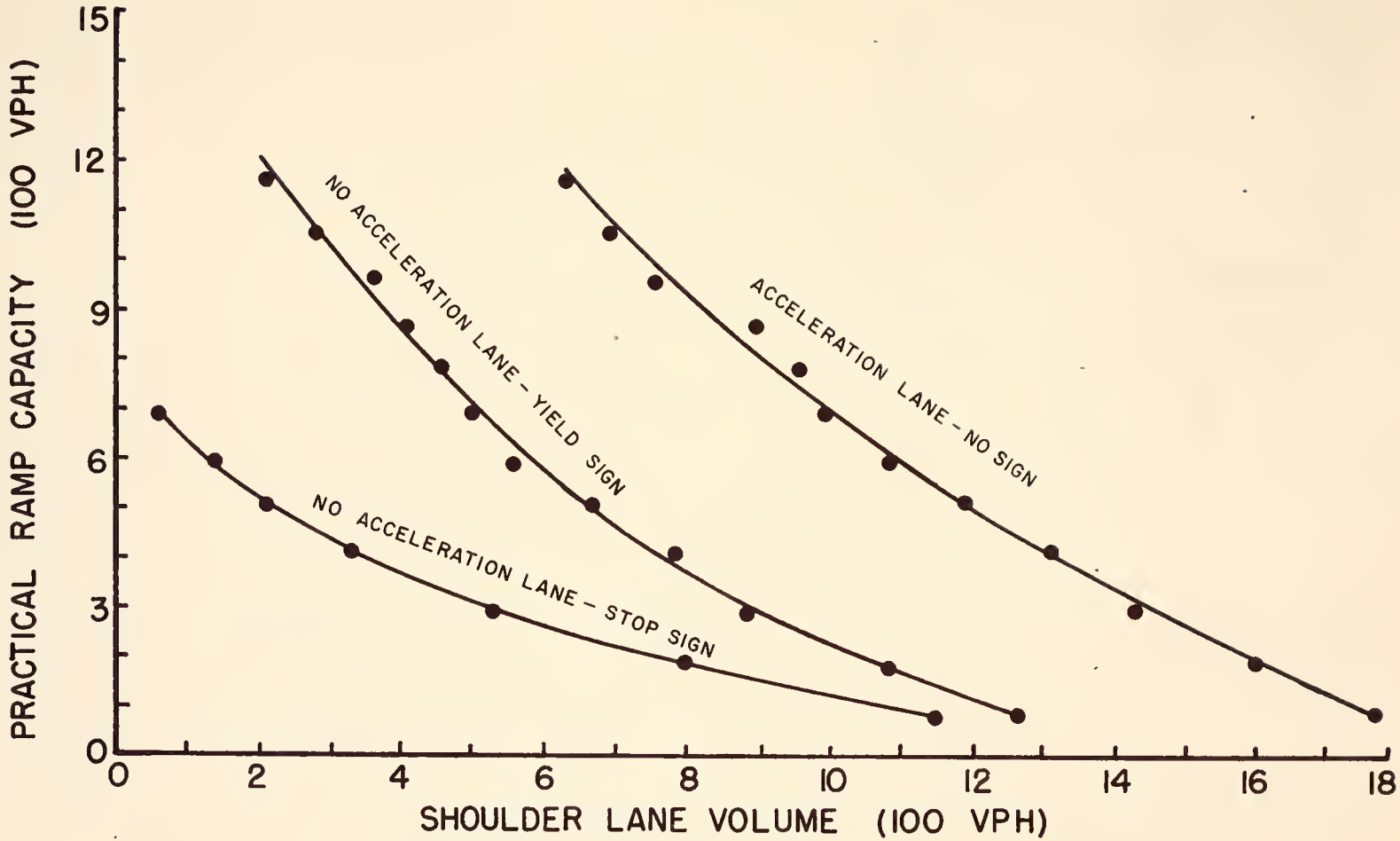


FIG. 14 - PRACTICAL CAPACITIES OF FREEWAY ON-RAMPS

TABLE 14

SUMMARY OF LEAST-SQUARE EQUATIONS FOR PREDICTING PRACTICAL CAPACITIES OF FREEWAY ON-RAMPS

Prediction Model: $y = e^{(a+bx+cx^2)}$*

Regression Coefficients			Statistical Characteristics			
			Limits of Analysis-x			
a	b	c	Low	High	R^2	Number of Observations
No Acceleration Lane and Stop-Sign Control						
+6.5781	-0.0014546	-0.0000002356	63	1150	.986	7
No Acceleration Lane and Yield-Sign Control						
+7.2901	-0.0009507	-0.0000009388	203	1267	.996	12
Acceleration Lane and No Sign Control						
+6.9814	+0.0008034	-0.0000012041	633	1783	.996	12

* y = Practical Capacity -- vph
x = Shoulder-Lane Volume -- vph

The various queue-length estimates that were obtained are plotted as functions of shoulder-lane volume in Figures 15, 16, and 17, respectively, for ramps with no acceleration lane and stop-sign control, no acceleration lane and yield-sign control, and an acceleration with no sign control. Although there was relatively little scatter in the data describing the queuing conditions on the stop-sign controlled ramp, a statistical analysis was performed for the purpose of deriving prediction models. An empirical equation of the form,

$$y = e^{(a + bx)},$$

was fitted to the data describing average queue lengths using the method of least squares. Equations of the form,

$$y = e^{\frac{1}{a + bx + cx^2}},$$

were derived for prediction of 85th, 90th and 95th percentile-queue lengths. The results of the statistical analyses are presented in Table 15, where it should be noted that the multiple R^2's (r^2 in the case of the average queue-length model) for the transformed equations were all equal to or greater than 0.967.

The various practical-capacity queuing characteristics for the ramp with yield-sign control and the ramp with an acceleration lane were described by empirical, least-square equations of the form,

$$y = e^{(a + bx)},$$

and the results are presented in Table 16. Because of scatter in the average queue length data the r^2 values were only 0.916 and 0.883 for the yield-sign control condition and the acceleration-lane condition, respectively.

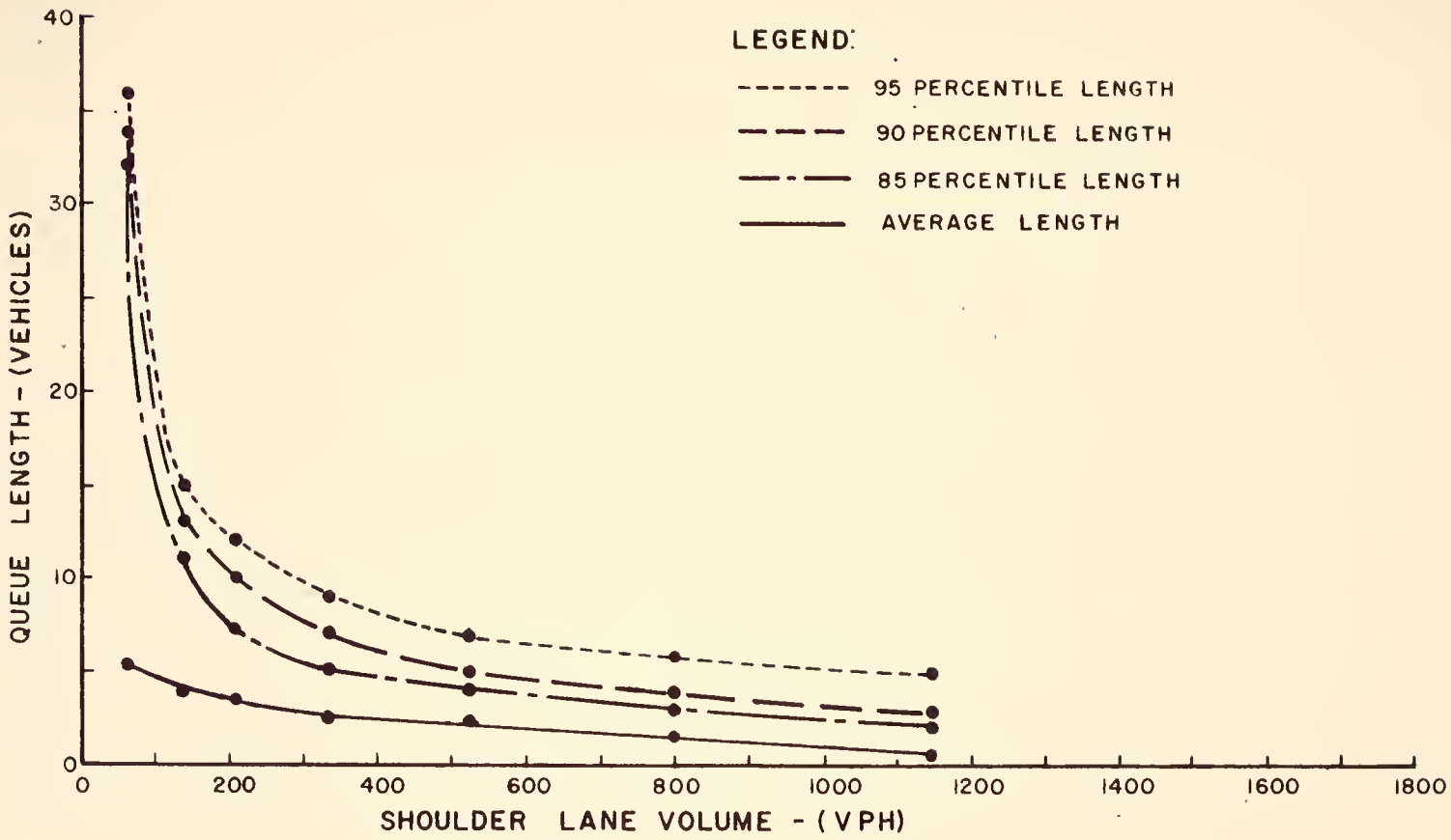


FIG. 15 - QUEUE LENGTHS AT PRACTICAL CAPACITY WITH STOP-SIGN CONTROL

TABLE 15

SUMMARY OF LEAST-SQUARE EQUATIONS FOR PREDICTING QUEUE CHARACTERISTICS UNDER PRACTICAL CAPACITY CONDITIONS ON ON-RAMPS WITH NO ACCELERATION LANE AND STOP-SIGN CONTROL

Average Queue Prediction Model: $y = e^{(a+bx)}$ *

Percentile Queue Prediction Model: $y = e^{(1/(a+bx+cx^2))}$ *

Variable Predicted	Regression Coefficients			Statistical Characteristics			
				Limits of Analysis-x			
	a	b	c	Low	High	R^2	Number of Observations
Avg. Queue	+1.7006	-0.0017448		63	1150	.967(r^2)	7
85% Queue	+0.3214	+0.0006068	+0.0000002942	63	1150	.976	7
90% Queue	+0.2714	+0.0007434	-0.0000001732	63	1150	.988	7
95% Queue	+0.2738	+0.0005952	-0.0000002623	63	1150	.972	7

* y = Queue Variable Predicted -- vehicles
x = Shoulder-Lane Volume -- vph

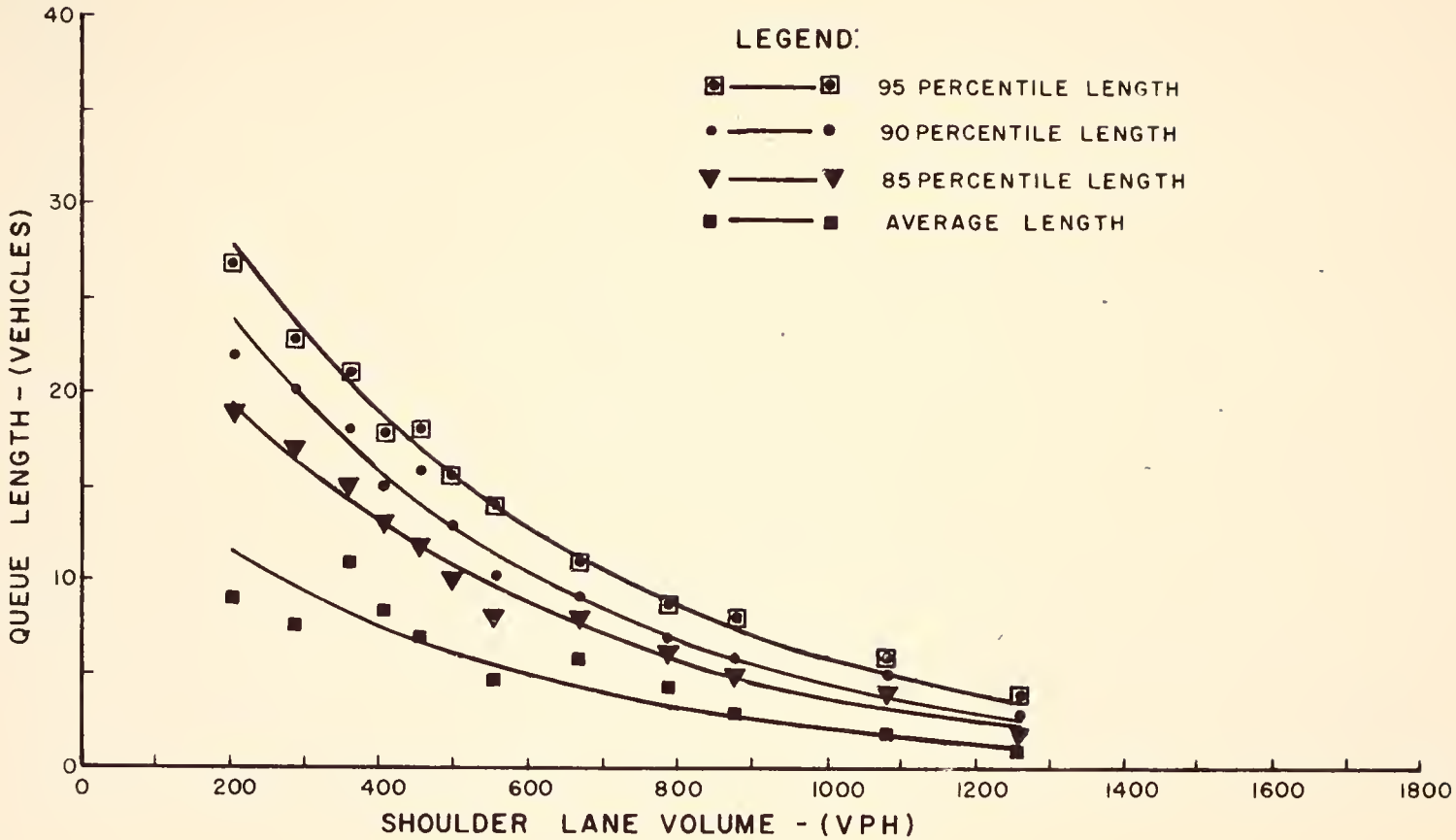


FIG. 16 - QUEUE LENGTHS AT PRACTICAL CAPACITY WITH YIELD-SIGN CONTROL

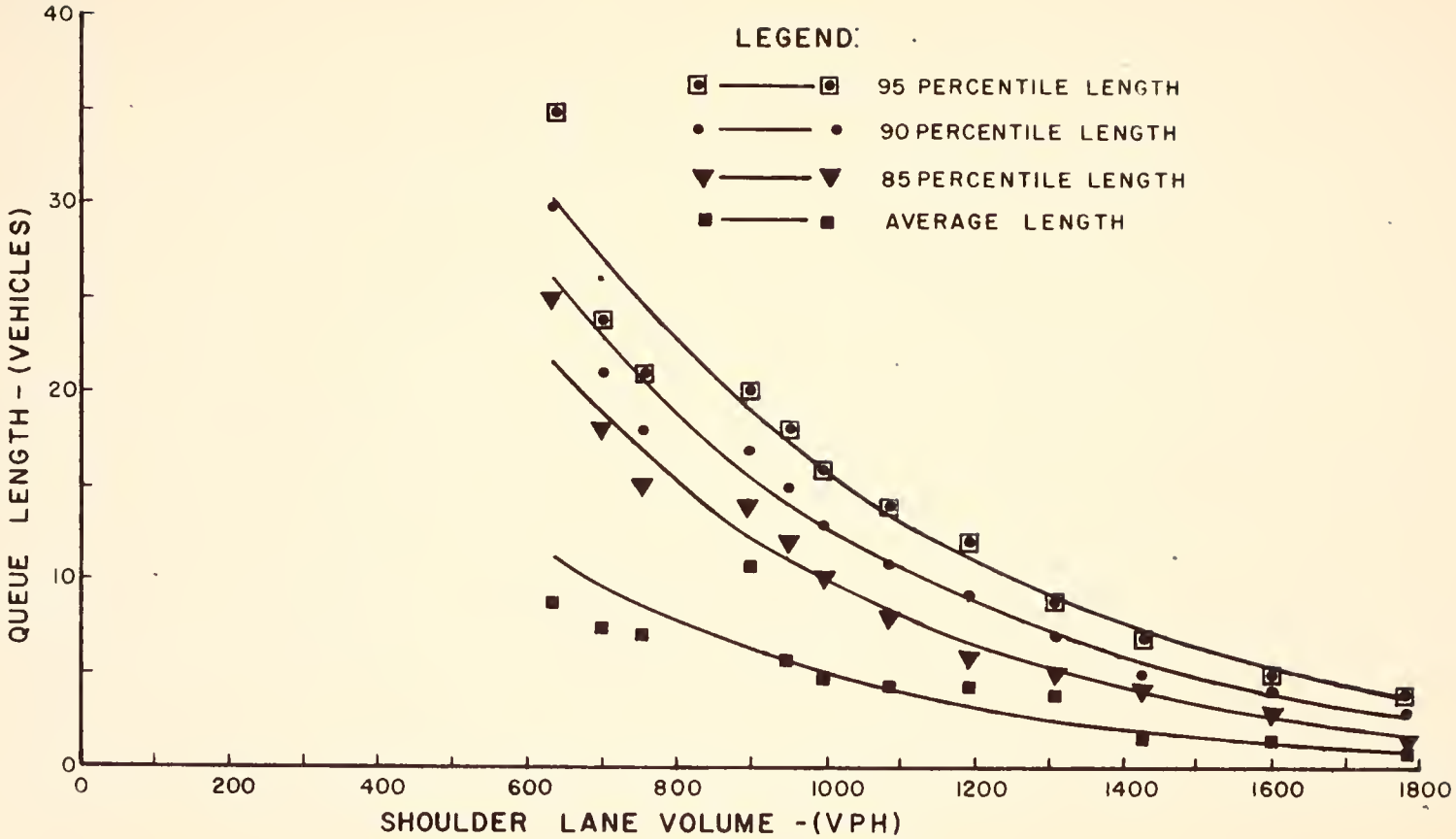


FIG.17- QUEUE LENGTHS AT PRACTICAL CAPACITY WITH ACCELERATION LANE

TABLE 16

SUMMARY OF LEAST-SQUARE EQUATIONS FOR PREDICTING QUEUE CHARACTERISTICS UNDER PRACTICAL CAPACITY CONDITIONS ON ON-RAMPS WITH NO ACCELERATION AND YIELD-SIGN CONTROL AND ON ON-RAMPS WITH AN ACCELERATION LANE AND NO SIGN CONTROL

Prediction Model: $y = e^{(a+bx)}$ *

Variable Predicted	Regression Coefficients a	Regression Coefficients b	Statistical Characteristics: Limits of Analysis-x Low	Limits of Analysis-x High	r^2	Number of Observations
No Acceleration Lane and Yield-Sign Control						
Avg. Queue	+2.8634	-0.0021425	203	1267	.916	12
85% Queue	+3.3754	-0.0020187	203	1267	.980	12
90% Queue	+3.6052	-0.0021090	203	1267	.964	12
95% Queue	+3.7252	-0.0019438	203	1267	.984	12
Acceleration Lane and No Sign Control						
Avg. Queue	+3.7821	-0.0021669	633	1783	.883	12
85% Queue	+4.4148	-0.0021100	633	1783	.988	12
90% Queue	+4.4861	-0.0019356	633	1783	.986	12
95% Queue	+4.5476	-0.0017941	633	1783	.983	12

* y = Queue Variable Predicted -- vehicles
x = Shoulder-Lane Volume -- vph

Ramp Capacities By Other Delay Criteria

In Figures 18, 19, and 20 the practical-capacity relationships obtained from analyses of the ramp and shoulder-lane volume conditions that permit only 85 percent of the ramp vehicles to leave the ramp without incurring delay in excess of 60 seconds, are compared to several other capacity relationships described by volume conditions that generate various levels of average delay. In general, practical capacities based on the proposed practical-capacity definitions are similar to the capacities that could be realized with average delays in the vicinity of 30 seconds.

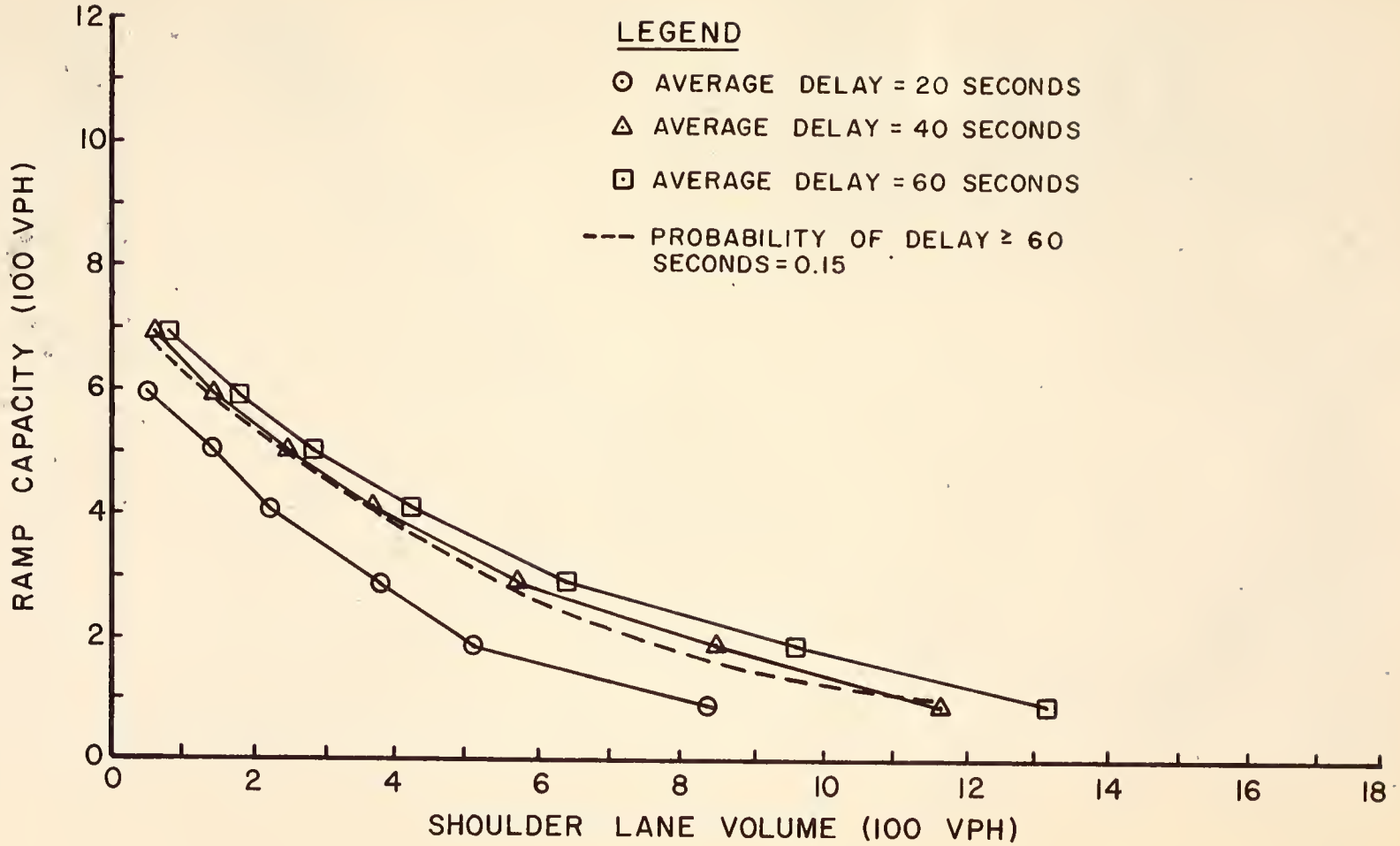


FIG. 18 - COMPARISON OF CAPACITIES DEFINED BY SEVERAL DELAY CRITERIA FOR ON-RAMPS WITH NO ACCELERATION LANE AND STOP-SIGN CONTROL

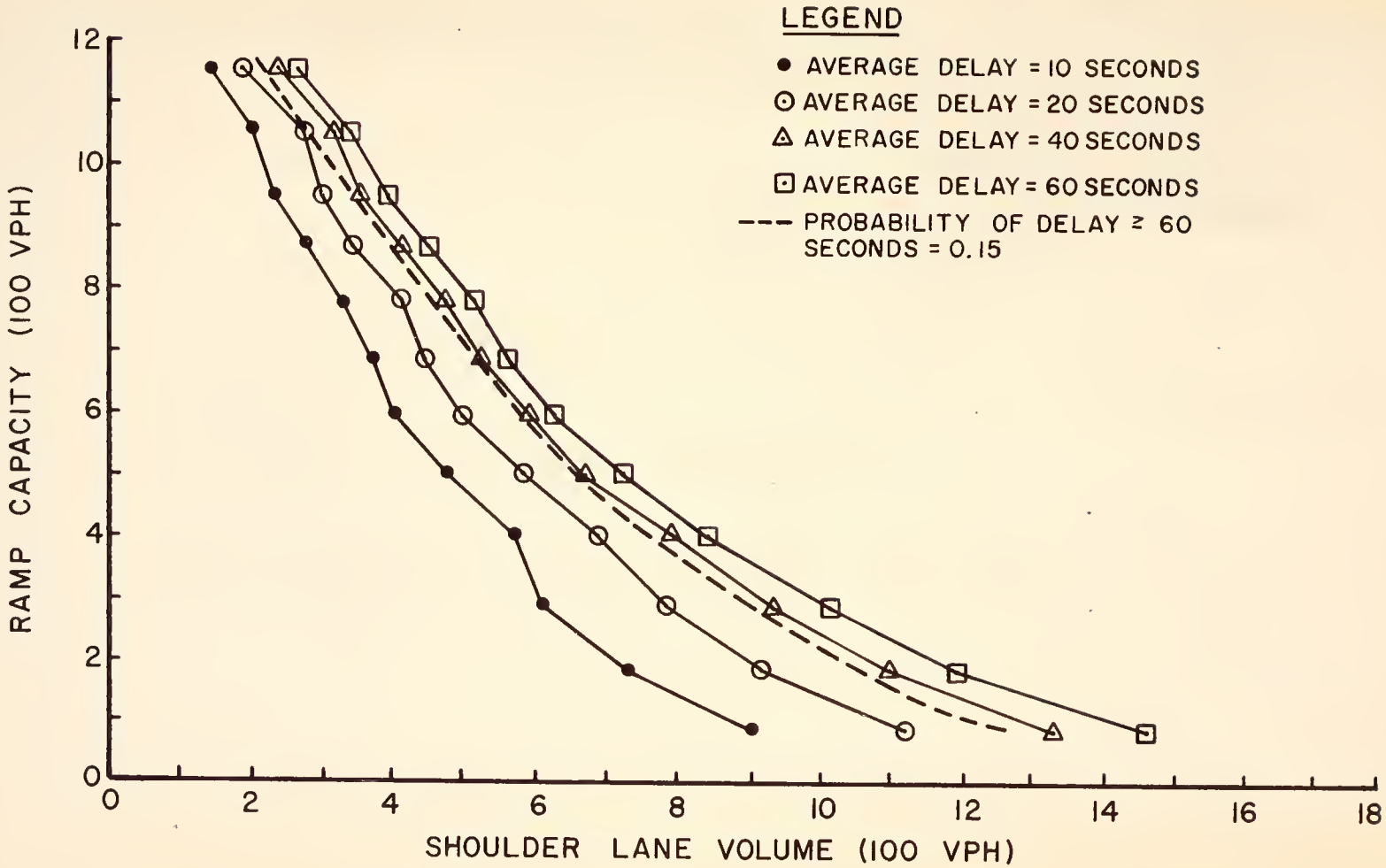


FIG 19- COMPARISON OF CAPACITIES DEFINED BY SEVERAL DELAY CRITERIA FOR ON-RAMPS WITH NO ACCELERATION LANE AND YIELD-SIGN CONTROL

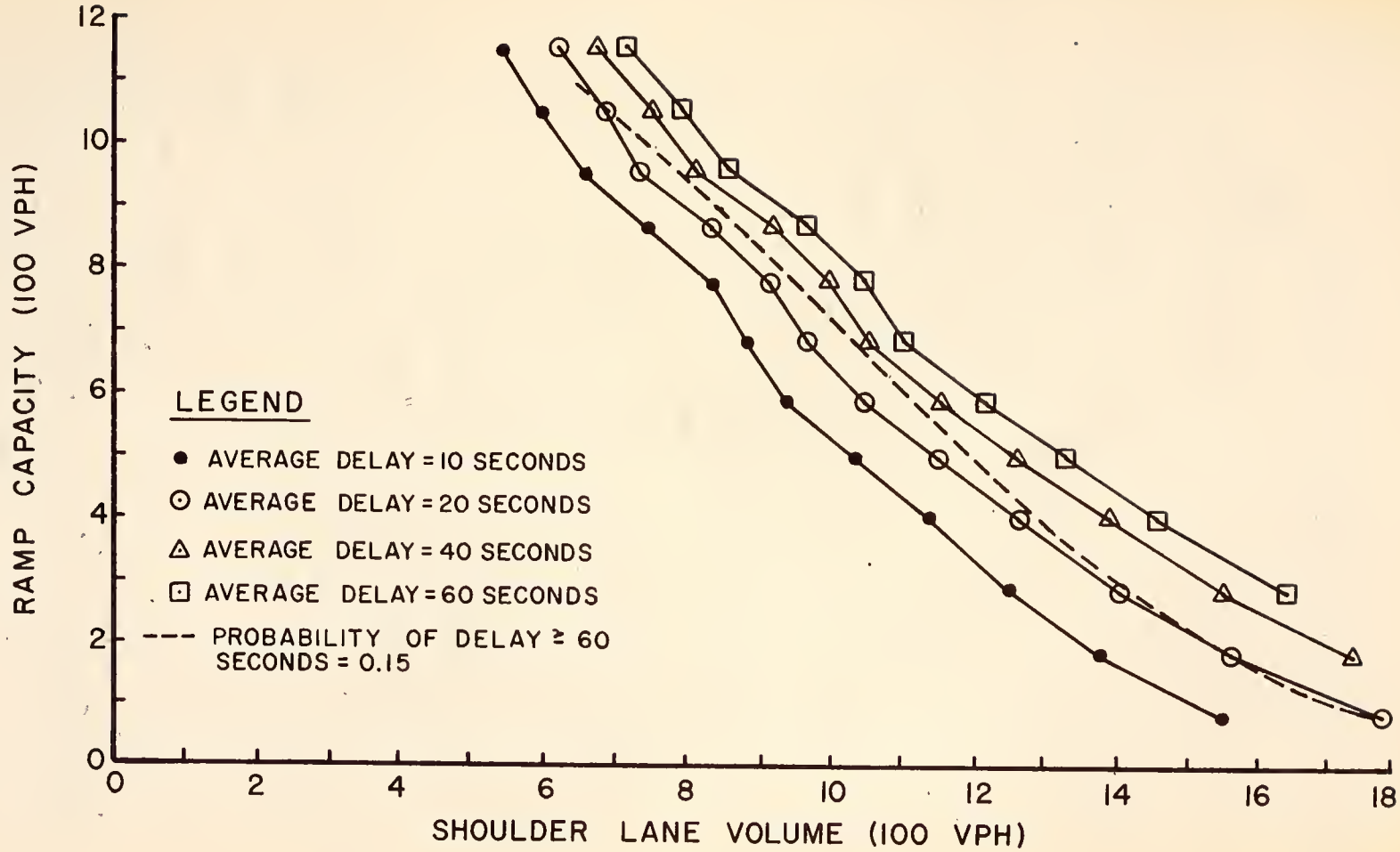


FIG. 20 - COMPARISON OF CAPACITIES DEFINED BY SEVERAL DELAY CRITERIA FOR ON-RAMPS WITH AN ACCELERATION LANE AND NO SIGN CONTROL

Comparison of Possible and Practical Capacities

A comparison of the practical and possible capacities defined in the study revealed that the practical capacities of stop-sign and yield-sign controlled ramps vary from about 30 percent of possible capacity at high shoulder-lane volumes, to about 90 percent of possible capacity at low shoulder-lane volumes. In the case of the ramp with an acceleration lane, the comparison indicated that practical capacity varied from approximately 50 percent of possible at high shoulder-lane volumes to nearly 110 percent of possible capacity at the lowest shoulder-lane volume studied. It is not reasonable for practical capacity to exceed possible capacity; but the discrepancy can be explained.

This discrepancy in the results obtained from the queuing model and the simulator resulted from an operating condition that was assumed to exist and was built into the queuing model. The assumed condition did not always materialize, however, when the possible capacity levels predicted by the queuing model were reproduced by the simulator. The operating restriction that was imposed by the queuing model required every queue of ramp vehicles utilizing the capacity available in each single shoulder-lane gap, to depart from a stopped condition. As a consequence the first driver in each queue based his gap-acceptance decision on the more restrictive gap-acceptance model for stopped, first-in-line vehicles. The simulator, however, did not absolutely require that every queue depart from a stopped condition. Even as possible capacity was approached, the driver had the option to modify his speed in the ramp-acceleration lane area in such a manner as to delay his arrival time at the entry point to the shoulder lane. By effecting this moving

delay the driver was able to take advantage of the less restrictive, no-stop, gap-acceptance model. In real life, drivers undoubtedly follow this practice. The results suggest that the possible capacities of ramps with acceleration lanes and no sign control are actually a little higher than indicated by the queuing model.

Although there were no instances in which the practical capacities exceeded the possible capacities of ramps with no acceleration lanes and yield-sign control, the discrepancies in the above case led to an investigation of the adequacy of the assumed operating conditions for this latter type of ramp. A review of the vehicle-counter logs kept by the yield-sign simulator monitor revealed, however, that even though absolute stops were not required with yield control, the maneuver distance available without an acceleration lane was not long enough to permit moving delays. As a consequence the necessary stop condition was forced by the traffic itself at those volume levels approaching possible capacity as defined by the queuing model.

Of course, there is no question as to whether or not the necessary stop condition is satisfied on ramps with no acceleration lanes and stop-sign control. A stop is mandatory for every single vehicle leaving the ramp when this type of sign control is utilized.

SUMMARY AND CONCLUSIONS

1. Objective criteria were established for the measurement of the possible and practical capacities of freeway on-ramps. These criteria were stated in definition form.
   a. Possible capacity of a freeway on-ramp is the maximum number of vehicles that can enter the through highway during one hour under the prevailing conditions

with a continual backlog of waiting vehicles.

b. Practical capacity of a freeway on-ramp is the maximum number of vehicles that can enter the through highway during one hour with 85 percent of the drivers being able to leave the ramp without being delayed more than 60 seconds.

2. The micro aspects of freeway on-ramp areas and their traffic were modeled in the mathematical mode, within the present understanding of traffic flow theory. In some cases, empirical estimates were substituted for presently undefined functional relationships.

3. Rules of operation were established for the on-ramp area that provided a framework within which the models describing micro-aspects were assembled as functional systems.

a. The rules for operation of the ramp system at possible capacity were implied by the definition of possible capacity. These rules provided for the development of a deterministic queuing model that adequately describes the possible capacity of ramps with no acceleration lane and either stop- or yield-sign control. This queuing model predicted possible capacities that were slightly low in the case of ramps with an acceleration lane and no sign control.

b. More general rules were designed and implemented as control mechanisms in a computer-oriented ramp simulator. A wide range of ramp and shoulder-lane volume combinations were realistically generated by this model. Traffic monitors constructed as integral parts of the simulator measured and recorded several indices of traffic performance. The most important of these were average and percentile queue lengths,

average delay, and the probability that delay exceeds 60 seconds.

4. Statistical models were derived to define the various indices of performance as functions of ramp and shoulder-lane volume conditions for each type of ramp design considered.

5. Practical capacities were defined by obtaining solutions to the empirically derived models describing the probability of delay in excess of 60 seconds. Ramp and shoulder-lane volume combinations that generated a probability of 0.15 constituted a practical capacity situation.

6. The average queue models were solved and percentile queue data were evaluated at practical capacity volume conditions to obtain ramp storage requirements for ramps operating at practical capacity.

7. The results obtained from the queuing and simulation analyses can be extremely useful in the design of new on-ramp facilities and in evaluating the adequacy of existing facilities. The procedure for applying these results to a particular ramp situation involves two steps:

   a. Obtain an estimate of the amount of traffic that is using, or is expected to use, the shoulder lane. This may be done by actual field study or by using Hess's lane-distribution models given in the section entitled "Descriptors of the Ramp Situation."

   b. Obtain possible capacity, practical capacity, and performance characteristics associated with practical capacity from the appropriate models derived in this study.

8. Monte Carlo simulation is a useful, practical, and efficient technique for studying freeway on-ramp operations. The proposed simulator

required approximately two minutes for each combination of ramp and shoulder-lane volumes that was simulated. Although constant sample sizes of 1000 ramp vehicles were observed on each run, variations in the ramp flow-rates resulted in variation of the real time/computer time ratio. These ratios ranged from 360/1 to 30/1. Approximately one-half of the computer time was spent pre-loading the ramp system, preparing statistical summaries of the results, and writing the simulation report.

LIST OF REFERENCES

1. American Association of State Highway Officials, A Policy On The Geometric Design of Rural Highways, Washington: 1954

2. Committee on Highway Capacity, Highway Capacity Manual, U. S. Dept. of Commerce, Bureau of Public Roads, Washington: 1950.

3. Gerlough, D. L., "Traffic Inputs for Simulation on a Digital Computer," Proceedings, Highway Research Board, Vol. 38, Washington: 1959, pp. 480-492.

4. Hess, Joseph W., "Capacities and Characteristics of Ramp-Freeway Connections," Highway Research Board, Record No. 27, Washington: 1963, pp. 69-115.

5. IBM 7090/7094 Programming System, Reference Manual C28-6274-1, International Business Machine Corporation, White Plains, New York, 1963.

6. Kell, J. H., "A Theory of Traffic Flow on Urban Streets," Proceedings, The 13th Annual Western Section Meeting of the Institute of Traffic Engineers, San Francisco, California, 1960, pp. 66-70.

7. Levy, S. L., Aaron Glickstein, and Leon D. Findley, "A Study of the Application of Computer Simulation Techniques to Interchange Design Problems," Final Report, Midwest Research Institute, Missouri, 1960.

8. Levy, Sheldon L., and Philip A. Perchonok, "Application of Digital Simulation Techniques to Freeway On-Ramp Traffic Operations," Proceedings, Highway Research Board, Vol. 39, Washington: 1960, pp. 506-523.

9. Levy, Sheldon L., and Aaron Glickstein, "Application of Digital Simulation Techniques to Highway Design Problems," Paper presented at the Western Joint Computer Conference, Los Angeles, California, 1961.

10. Levy, Sheldon L., and Philip A. Perchonok, "Ramp Simulation Techniques on a Digital Computer," Highway Research Board, Bulletin No. 293, Washington: 1961.

11. Matson, T. M., W. S. Smith, and F. W. Hurd, Traffic Engineering, McGraw-Hill Book Companry, New York, 1955.

12. May, D., Jr., "Traffic Characteristics and Phenomena on High Density Controlled Access Facilities." Traffic Engineering 31: 1-19, 56 March 1961.

13. Moskowitz, Karl and Leonard Newman, "Notes on Freeway Capacity," Highway Research Board, Record No. 27, Washington: 1963, pp. 44-68.

14. Moskowitz, Karl "Research on Operating Characteristics of Freeways," Proceedings, Institute of Traffic Engineers, pp. 85-110, 1956.

15. Oliver, Robert M. Farnsworth Bisbee, Gerold Pestalozzi, and Jean-Claude Dysli, "Vehicles Queues in Mixed Traffic Streams," Progress Report, Institute of Transportation and Traffic Engineering, Report No. 35, California: 1962.

16. Pearson, R. H., and M. G. Ferreri, "Operational Study--Schuylkill Expressway," Highway Research Board, Bulletin No. 293, Washington: 1961, pp. 104-123.

17. Schuhl, A., "The Probability Theory Applied to Distribution of Vehicles on Two-Lane Highways," Poisson and Traffic, Eno Foundation for Highway Traffic Control, Saugatuck, Conn., 1955, pp. 59-75.

18. Weiss, G. M., and A. A. Maradudin, "Some Problems in Traffic Delay," Operations Research, Vol. 10, No. 1, Baltimore: 1962, pp. 74-141.

19. Wohl, Martin, "Simulation--Its Application to Traffic Engineering, Part I," Traffic Engineering, Vol. 30, No. 11, August, 1960, pp. 13-17, 29.

20. Wohl, M. "Simulation--Its Application to Traffic Engineering, Part II," Traffic Engineering, Vol. 31, No. 11, pp. 19-25, 56, October, 1960.